

Contents

1	Introduction	1
1.1	Preview	2
1.2	Prerequisites	2
1.3	Numbering	2
2	Gambling Houses and the Conservation of Fairness	5
2.1	Introduction	5
2.2	Gambles, Gambling Houses, and Strategies	6
2.3	Stopping Times and Stop Rules	9
2.4	An Optional Sampling Theorem	11
2.5	Martingale Convergence Theorems	15
2.6	The Ordinals and Transfinite Induction	15
2.7	Uncountable State Spaces and Continuous-Time	18
2.8	Problems for Chapter 2	19
3	Leavable Gambling Problems	23
3.1	The Fundamental Theorem	24
3.2	The One-Day Operator and the Optimality Equation	26
3.3	The Utility of a Strategy	27
3.4	Some Examples	30
3.5	Optimal Strategies	42
3.6	Backward Induction: An Algorithm for U	48
3.7	Problems for Chapter 3	52

4 Nonleavable Gambling Problems	59
4.1 Introduction	59
4.2 Understanding $u(\sigma)$	60
4.3 A Characterization of V	68
4.4 The Optimality Equation for V	69
4.5 Proving Optimality	70
4.6 Some Examples	70
4.7 Optimal Strategies	75
4.8 Another Characterization of V	78
4.9 An Algorithm for V	82
4.10 Problems for Chapter 4	84
5 Stationary Families of Strategies	89
5.1 Introduction	89
5.2 Comparing Strategies	90
5.3 Finite Gambling Problems	94
5.4 Nonnegative Stop-or-Go Problems	96
5.5 Leavable Houses	101
5.6 An Example of Blackwell and Ramakrishnan	106
5.7 Markov Families of Strategies	109
5.8 Stationary Plans in Dynamic Programming	109
5.9 Problems for Chapter 5	110
6 Approximation Theorems	113
6.1 Introduction	113
6.2 Analytic Sets	114
6.3 Optimality Equations	124
6.4 Special Cases of Theorem 1.2	128
6.5 The Going-Up Property of \overline{M}	139
6.6 Dynamic Capacities and the Proof of Theorem 1.2	144
6.7 Approximating Functions	150
6.8 Composition Closure and Saturated House	158
6.9 Problems for Chapter 6	165
7 Stochastic Games	171
7.1 Introduction	171
7.2 Two-Person, Zero-Sum Games	172
7.3 The Dynamics of Stochastic Games	176
7.4 Stochastic Games with \limsup Payoff	179
7.5 Other Payoff Functions	180
7.6 The One-Day Operator	181
7.7 Leavable Games	184
7.8 Families of Optimal Strategies for Leavable Games	189
7.9 Examples of Leavable Games	191
7.10 A Modification of Leavable Games and the Operator T	196

7.11 An Algorithm for the Value of a Nonleavable Game	198
7.12 The Optimality Equation for V	201
7.13 Good Strategies in Nonleavable Games	203
7.14 Win, Lose, or Draw	207
7.15 Recursive Matrix Games	210
7.16 Games of Survival	212
7.17 The Big Match	216
7.18 Problems for Chapter 7	221
References	227
Symbol Index	239
Index	241