Contents

Prefact	ce	<i>page</i> xiii
Acknown	pwledgments	xvii
1	Formulation of physical problems	1
1.1	Transverse vibration of a taut string	1
1.2	Longitudinal vibration of an elastic rod	4
1.3	Traffic flow on a freeway	6
1.4	Seepage flow through a porous medium	7
1.5	Diffusion in a stationary medium	10
1.6	Shallow water waves and linearization	12
	1.6.1 Nonlinear governing equations	12
	1.6.2 Linearization for small amplitude	15
2	Classification of equations with two independ	lent
	variables	20
2.1	A first-order equation	20
2.2	System of first-order equations	22
2.3	Linear second-order equations	25
	2.3.1 Constant coefficients	25
	2.3.2 Variable coefficients	28
3	One-dimensional waves	33
3.1	Waves due to initial disturbances	33
3.2	Reflection from the fixed end of a string	38
3.3	Specification of initial and boundary data	40
3.4	Forced waves in a long string	41
3.5	Uniqueness of the Cauchy problem	44
3.6	Traffic flow - a taste of nonlinearity	45
3.7	Green light at the head of traffic	46
3.8	Traffic congestion and jam	50

viii Contents

4	Finite domains and separation of variables	57
4.1	Separation of variables	57
4.2	One-dimensional diffusion	63
4.3	Eigenfunctions and base vectors	65
4.4	Partially insulated slab	66
4.5	Sturm-Liouville problems	71
4.6	Steady forcing	75
4.7	Transient forcing	76
4.8	Two-dimensional diffusion	78
4.9	Cylindrical polar coordinates	80
4.10	Steady heat conduction in a circle	82
5	Elements of Fourier series	91
5.1	General Fourier series	91
5.2	Trigonometric Fourier series	93
	5.2.1 Full Fourier series	93
	5.2.2 Fourier cosine and sine series	95
	5.2.3 Other intervals	96
5.3	Exponential Fourier series	98
5.4	Convergence of Fourier series	99
6	Introduction to Green's functions	105
6.1	The δ function	105
6.2	Static deflection of a string under a concentrated load	108
6.3	String under a simple harmonic point load	110
6.4	Sturm-Liouville boundary-value problem	112
6.5	Bending of an elastic beam on an elastic foundation	114
	6.5.1 Formulation of the beam problem	114
	6.5.2 Beam under a sinusoidal concentrated load	117
6.6	Fundamental solutions	121
6.7	Green's function in a finite domain	124
6.8	Adjoint operator and Green's function	125
7	Unbounded domains and Fourier transforms	132
7.1	Exponential Fourier transform	132
	7.1.1 From Fourier series to Fourier transform	132
	7.1.2 Transforms of derivatives	134
	7.1.3 Convolution theorem	134
7.2	One-dimensional diffusion	135
	7.2.1 General solution in integral form	135
	7.2.2 A localized source	137
	7.2.3 Discontinuous initial temperature	139

Contents	ix

7.3	Forced waves in one dimension	141
7.4	Seepage flow into a line drain	143
7.5	Surface load on an elastic ground	145
	7.5.1 Field equations for plane elasticity	145
	7.5.2 Half plane under surface load	147
	7.5.3 Response to a line load	149
7.6	Fourier sine and cosine transforms	153
7.7	Diffusion in a semi-infinite domain	154
7.8	Potential problem in a semi-infinite strip	159
8	Bessel functions and circular boundaries	165
8.1	Circular region and Bessel's equation	165
8.2	Bessel function of the first kind	167
8.3	Bessel function of the second kind for integer order	171
8.4	Some properties of Bessel functions	174
	8.4.1 Recurrence relations	175
	8.4.2 Behavior for small argument	176
	8.4.3 Behavior for large argument	176
	8.4.4 Wronskians	178
	8.4.5 Partial wave expansion	179
8.5	Oscillations in a circular region	180
	8.5.1 Radial eigenfunctions and natural modes	180
	8.5.2 Orthogonality of natural modes	182
	8.5.3 Transient oscillations in a circular pond	184
8.6	Hankel functions and wave propagation	185
	8.6.1 Wave radiation from a circular cylinder	186
	8.6.2 Scattering of plane waves by a circular cylinder	190
8.7	Modified Bessel functions	192
8.8	Bessel functions with complex argument	194
8.9	Pipe flow through a vertical thermal gradient	195
	8.9.1 Formulation	195
	8.9.2 Solution for rising ambient temperature	199
8.10	Differential equations reducible to Bessel form	201
9	Complex variables	210
9.1	Complex numbers	210
9.2	Complex functions	212
9.3	Branch cuts and Riemann surfaces	215
9.4	Analytic functions	220
9.5	Plane seepage flows in porous media	223
9.6	Plane flow of a perfect fluid	225

x Contents

9.7	Simple irrotational flows	227
9.8	Cauchy's theorem	229
9.9	Cauchy's integral formula and inequality	235
9.10	Liouville's theorem	237
9.11	Singularities	238
9.12	Evaluation of integrals by Cauchy's theorems	239
9.13	Jordan's lemma	246
9.14	Forced harmonic waves and the radiation condition	247
9.15	Taylor and Laurent series	250
9.16	More on contour integration	254
10	Laplace transform and initial value problems	260
10.1	The Laplace transform	260
10.2	Derivatives and the convolution theorem	263
10.3	Coupled pendula	265
10.4	One-dimensional diffusion in a strip	267
10.5	A string-oscillator system	269
10.6	Diffusion by sudden heating at the boundary	272
10.7	Sound diffraction near a shadow edge	275
10.8	*Temperature in a layer of accumulating snow	280
11	Conformal mapping and hydrodynamics	289
11.1	What is conformal mapping?	289
11.2	Relevance to plane potential flows	290
11.3	Schwarz-Christoffel transformation	291
11.4	An infinite channel	295
	11.4.1 Mapping onto a half plane	295
	11.4.2 Source in an infinite channel	296
11.5	A semi-infinite channel	298
11.6	An estuary	301
11.7	Seepage flow under an impervious dam	302
11.8	Water table above an underground line source	307
12	Riemann-Hilbert problems in hydrodynamics	
	and elasticity	318
12.1	Riemann-Hilbert problem and Plemelj's formulas	318
12.2	Solution to the Ricmann-Hilbert problem	320
12.3	Linearized theory of cavity flow	322
12.4	Schwarz's principle of reflection	327
12.5	*Complex formulation of plane elasticity	330
	12.5.1 Airy's stress function	330
	12.5.2 Stress components	331

	12.5.3 Displacement components	332
	12.5.4 Half-plane problems	333
12.6	*A strip footing on the ground surface	335
	12.6.1 General solution to the boundary-value problem	335
	12.6.2 Vertically pressed flat footing	336
13	Perturbation methods – the art of	
	approximation	343
13.1	Introduction	343
13.2	Algebraic equations	345
	13.2.1 Regular perturbations	345
	13.2.2 Singular perturbations	347
13.3	Parallel flow with heat dissipation	349
13.4	Freezing of water surface	352
13.5	Method of multiple scales for an oscillator	358
	13.5.1 Weakly damped harmonic oscillator	358
	13.5.2 Elastic spring with weak nonlinearity	363
13.6	Theory of homogenization	367
	13.6.1 Differential equation with periodic coefficient	367
	13.6.2 *Darcy's law in seepage flow	370
13.7	*Envelope of a propagating wave	376
13.8	Boundary-layer technique	381
13.9	Seepage flow in an aquifer with slowly varying depth	384
13.10	Water table near a cracked sheet pile	390
13.11	*Vibration of a soil layer	395
	13.11.1 Formulation	395
	13.11.2 The outer solution	397
	13.11.3 The boundary-layer correction	399
14	Computer algebra for perturbation analysis	408
14.1	Getting started	408
14.2	Algebraic and trigonometric operations	409
	14.2.1 Elementary operations	409
	14.2.2 Functions	411
	14.2.3 Algebraic reductions	412
	14.2.4 Trigonometric reductions	413
	14.2.5 Substitutions and manipulations	414
14.3	Exact and perturbation methods for algebraic equations	417
14.4	Calculus	421
	14.4.1 Differentiation	421
	14.4.2 Integration	423

Contents
(

14.5	Ordinary differential equations	426
14.6	Pipe flow in a vertical thermal gradient	427
14.7	Duffing problem by multiple scales	435
14.8	Evolution of wave envelope on a nonlinear string	441
Appendices		447
Bibliography		453
Index		457