CONTENTS

PREFACE ix

CHAPTER 1 INTRODUCTORY CONCEPTS 1
1.1 Introduction 1
1.2 Definition of Terms 1
1.3 Elimination of Arbitrary Constants; Primitives 6
1.4 Formulation of Differential Equations 8

CHAPTER 2 DIFFERENTIAL EQUATIONS OF FIRST ORDER AND FIRST DEGREE 21
2.1 Method of Separation of the Variables 21
2.2 The Integrating Factor 22
2.3 Homogeneity 25
2.4 Simple Substitutions 27
2.5 Exact Equations 29
2.6 Equations Reducible to Linear Form 32

CHAPTER 3 EQUATIONS OF FIRST ORDER AND NOT OF FIRST DEGREE 36
3.1 Introduction 36
3.2 Singular Solutions 36
3.3 Equations Solveable for the First Derivative 43
3.4 Equations Solveable for the Dependent Variable 45
3.5 Equations Solveable for the Independent Variable 46
3.6 Clairaut's Equation 47
3.7 D'Alembert's Equation 48
3.8 Picard's Method 49

CHAPTER 4 APPLICATIONS OF FIRST ORDER EQUATIONS 53
4.1 Geometric Applications and Orthogonal Trajectories 53
4.2 Heat Flow 58
4.3 Dynamics 60
4.4 Mixing of Solutions 62
4.5 Electrical Circuits 64
4.6 Growth and Decay 66
4.7 Viscous Flow in a Tube 67
4.8 Counterflow Heat Exchanger Mean Temperature Difference 70

CHAPTER 5 SECOND AND HIGHER ORDER DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS 76
5.1 The Differential Operator 76
5.2 Existence of Exponential Solutions 77
5.3 Completeness of Solutions and the Wronskian 79
5.4 Second Order Equations—Form of Solutions 81
5.5 Application—Mass, Spring, Damper System 86

CHAPTER 6 NON-HOMOGENEOUS SECOND AND HIGHER ORDER DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS 97
6.1 Existence of Complementary Function and Particular Integral 97
6.2 The Method of Undetermined Coefficients 98
6.3 Particular Integrals by Variation of Parameters 105
6.4 Application—Mass, Spring, Damper System 110
6.5 The Euler Equation 113

CHAPTER 7 APPLICATIONS OF N^{TH} ORDER EQUATIONS OF THE FIRST DEGREE 119
7.1 Beam Theory 119
7.2 Temperature Distribution in the Longitudinal Fin of Rectangular Profile 127
7.3 Temperature Distribution in the Longitudinal Fin of Parabolic Profile 131
7.4 The Catenary 135
7.5 The Buckling of Columns with Concentric or Axial Loads 137
7.6 Particle Motion 140
7.7 Motions of Planets and Kepler's Laws 143
7.8 The Accelerometer 146
7.9 Electrical Circuits 148
7.10 Mechanical Vibration 150
7.11 Analogs 155

CHAPTER 8 SIMULTANEOUS LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS 161
8.1 Characteristic Determinant and Arbitrary Constants 161
8.2 Procedure for Solution 164

CHAPTER 9 APPLICATIONS OF SIMULTANEOUS LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS 172
9.1 Electrical Circuits 172
9.2 Vibrations With n Degrees of Freedom 180
9.3 Servomechanisms 188

CHAPTER 10 EQUATIONS WITH VARIABLE COEFFICIENTS: INTEGRATION IN SERIES . . . 195
10.1 Infinite Series 195
10.2 Applicability of the Method of Frobenius 196
10.3 The Method of Frobenius 197
10.4 Legendre's Equation and Its Solutions 210
10.5 The Legendre Polynomials 216
10.6 Recurrence Relationship for The Legendre Polynomials 217
10.7 Rodrigues' Formula for the Legendre Polynomials 218
10.8 Legendre's Function of the Second Kind 219

CHAPTER 11 BESSEL AND GAMMA FUNCTIONS 222
11.1 Introduction 222
11.2 Recurrence Relationships 225
11.3 The Generalized Form of Bessel's Equation 229
11.4 The Gamma Function 233
11.5 Formal Solution To Bessel's Equation and the Bessel Function of the First Kind 238

11.6 Formal Solution for the Bessel Function of the Second Kind 242
11.7 Formal Solution for the Modified Bessel Functions of First and Second Kind 245
11.8 Derivation of Some of the Recurrence Relationships 247
11.9 Application—The Radial Fin of Rectangular Profile 250
11.10 Application—The Longitudinal Fin of Triangular Profile 251
11.11 Application—Pendulum with Variable Length 253

CHAPTER 12 NUMERICAL METHODS 257
12.1 Introduction 257
12.2 Mathematical Definition of the Problem 257
12.3 A First Approximation 258
12.4 Picard's Method 259
12.5 Runge's Method 261
12.6 The Runge-Kutta Method 264
12.7 Milne's Method 265
12.8 Finite Differences 268

CHAPTER 13 FUNCTIONS OF A COMPLEX VARIABLE 272
13.1 Introduction 272
13.2 Functions of a Complex Variable 275
13.3 Differentiation and Analytic Functions 277
13.4 Line Integrals 279
13.5 Taylor Series 284
13.6 Laurent Series 285
13.7 Singular Points 287
13.8 Residues 289
13.9 Improper Integrals 291

CHAPTER 14 THE LAPLACE TRANSFORMATION 297
14.1 Introduction 297
14.2 Definition of LaPlace Transform 297
14.3 The LaPlace Transform 299
14.4 The Inverse LaPlace Transform 300
14.5 Properties of LaPlace Transforms 302
14.6 Linear Differential Equations with Constant Coefficients 305
14.7 Simultaneous Linear Equations with Constant Coefficients 306
14.8 Partial Fraction Expansion of Rational Functions 310
14.9 The Inverse LaPlace Transform of a Rational Function 315
14.10 Application of the LaPlace Transform to the Solution of Linear Differential Equations 320

APPENDICES 329
A Hyperbolic Functions 329
B A Short Table of LaPlace Transforms 338
C Table of Gamma Functions 340
D Table of Bessel Functions 342
E Table of Legendre Polynomials 355
F The Roots of Equations 360
G A Compilation of Relationships from Algebra, Geometry, Trigonometry, Analytic Geometry and Calculus 365