

Preface	xiii
---------	------

---

<p><b>CHAPTER I Introduction, Featuring First-Order Linear Differential Equations</b></p> <p style="padding-left: 2em;">A. A Mixing Problem: The Polluted Lake</p> <p style="padding-left: 4em;">1. Deriving the Equation 2. Verifying the Role of the Derivative 3. Making the Prediction 4. Summary and Definition</p> <p style="padding-left: 2em;">B. Introduction to Initial Value Problems and General Solutions</p> <p style="padding-left: 4em;">1. Problems 2. Example 3. General Solution and Particular Solution 4. Initial Conditions 5. Sample Problems with Answers 6. Problems 7. Integrating a Discontinuous Function 8. Sample Problems with Answers 9. Problems</p> <p style="padding-left: 2em;">C. Obtaining Differential Equations from Simple Graphs</p> <p style="padding-left: 2em;">D. A Separation of Variables Method to Solve the Equation <math>y'(t) + a(t)y(t) = 0</math></p> <p style="padding-left: 4em;">1. The Method of Separation of Variables 2. The Uniqueness of the Solution 3. Recapitulation 4. Example 5. Definitions 6. Remarks on the Use of the Definite Integral 7. Example: the Integral in the Lake Problem 8. Sample Problems with Answers 9. Problems 10. Remarks on Direct Integration After Separation of Variables 11. Example 12. Example 13. Remarks on the</p>	<p>1</p> <p>2</p> <p>7</p> <p>16</p> <p>20</p>
--	--

E. Additional Models and Nonhomogeneous Equations	36
1. Cooling by Dilution 2. Cooling by Conduction: Newton's Law of Cooling 3. Example	
4. Comparison of Homogeneous and Nonhomogeneous Equations 5. Example	
6. Example: A Piecewise-Constant Nonhomogeneous Term 7. Sample Problems with Answers	
8. Problems 9. A Dynamics Problem: A Rocket Ascending Under Power 10. Problems	
F. An Integrating Factor for Solving Nonhomogeneous Equations	50
1. Method 2. Example: The Lake Problem in Case Some Pollutant Is Flowing into the Lake	
3. Example: Solution of the Rocket Problem from Section E 4. Problems	
G. More Complicated Models	56
1. Example: Friction on a Cable Wound About a Post 2. Example: Cable Friction Around a Horizontal Drum 3. Example: Mutual Heating and Cooling	
4. Example: Vehicle Response to Fluid Dynamic Control 5. Example: Machine with an Eccentric Rotor 6. Example: Hanging-Cable Problems	
7. Example: the Deflection of a Prismatic Beam	
H. Synopsis of Definitions and Results on Linear Differential Equations	77
I. Appendix: Graphing	82
1. Several Important Features of Graphs 2. The Graphs of Several Important Functions 3. The Graph of a Sum of Two Functions	
J. Appendix: Solving Nonlinear Equations by Separation of Variables	98
1. Examples and Comparison with Linear Equations 2. Outline of the Method of Separation of Variables 3. Sample Problems with Answers	
4. Problems	
K. Appendix: Numerical Solution Methods	108
1. Example: Euler's Method 2. The Method	
3. Reducing Step Size 4. Sample Problems with Answers 5. Problems 6. Smoothing	
7. A Predictor-Corrector Method 8. Sample Problems with Answers	

CHAPTER II Models of Engineering Systems 121

A. Electrical Model (Voltage-Current) 122  
1. Definition of Variables and Direction Conventions 2. Electrical Model Definition  
3. Principles for Using the Electrical Model  
4. Examples 5. Power and Energy in Electric Circuit Elements 6. Sample Problems with Answers 7. Problems

B. Mechanical Model (Distance-Force) 150  
1. Definition of Variables and Direction Conventions 2. Translational Mechanical Model Definition 3. Principles for Using the Mechanical Model 4. Examples 5. Power and Energy in Mechanical System Elements 6. Sample Problems with Answers 7. Problems

C. Translational Mechanical Force-Velocity Model 170  
1. The Model Equations and Interconnection Laws  
2. Examples 3. Sample Problems with Answers  
4. Problems

D. Eliminating Unwanted Variables from Systems of Equations 180  
1. Example 2. Sample Problems with Answers  
3. Problems 4. Initial Conditions 5. Example  
6. More Sample Problems with Answers  
7. Problems

---

CHAPTER III Transform Methods for Solving Linear Differential Equations with Constant Coefficients 191

THE LAPLACE TRANSFORM 192

A. Definition, Examples, and Method 192  
1. Example 2. Example 3. Example  
4. Method for Solving Initial Value Problems  
5. The Project 6. Transforms of Derivatives  
7. Sample Problems with Answers 8. Problems  
9. Remarks on the Validity of Laplace-Transform Formulas

B. Laplace-Transform Properties and a Short Table of Laplace Transforms 204  
1. Example:  $\mathcal{L}[\cos bt]$  and  $\mathcal{L}[\sin bt]$  2. The Property of Linearity 3. Example: Solving a

Second-Order Equation	4.	The Inverse Laplace Transform	
	5.	Sample Problems with Answers	
	6.	Problems	
	7.	The First Shifting Property	
	8.	Example: A Problem Whose Solution is a Damped Sinusoid	
	9.	Recognizing the Laplace Transforms of Damped Sinusoids	
	10.	Laplace Transforms of Powers of $t$	
	11.	Laplace Transforms of Integrals	
	12.	Differentiating a Laplace Transform	
	13.	Sample Problems with Answers	
	14.	Problems	
	15.	Making a Short Table of Laplace Transforms	
	16.	Initial Value Problems and the General Solution of a Differential Equation	
	17.	More Sample Problems with Answers	
	18.	More Problems	
C.		The Inverse Laplace Transform of a Rational Function	229
	1.	The Rational Function with Quadratic Denominator	
	2.	The General Form of the Partial-Fraction Decomposition	
	3.	Determining the Constants in the Partial-Fraction Decomposition	
	4.	Reducing the Number of Algebraic Equations to Be solved	
	5.	Finding Coefficients by Evaluating the Numerator	
	6.	Sample Problems with Answers	
	7.	Problems	
	8.	The Characteristic Polynomial for a Homogeneous Equation	
	9.	Sample Problems with Answers	
	10.	Problems	
D.		Step Functions and Their Applications	244
	1.	The Unit Step Function and Its Translates	
	2.	The Laplace Transform of a Step Function	
	3.	Example: Second Shifting Property	
	4.	Example	
	5.	Method and Example: Using the Second Shifting Theorem to Find and Graph an Inverse Laplace Transform	
	6.	Example	
	7.	Sample Problems with Answers	
	8.	Problems	
	9.	Method and Examples: Finding (Direct) Laplace Transforms Using the Second Shifting Theorem	
	10.	Checking the Solutions of Differential Equations	
	11.	Example: Excitation Over a Very Short Interval	
	12.	Sample Problems with Answers	
	13.	More Problems	
	14.	Comments on Piecewise-Continuous Functions	
E.		The Dirac $\delta$ and Its Uses	268
	1.	Example	
	2.	Our Interpretation of $\delta(t - t_1)$	
	3.	Example: Solving an Equation Containing $\delta(t - t_1)$	
	4.	Examples: Quantities and Processes Described by the Dirac $\delta$	
	5.	Example	
	6.	Sample Problems with Answers	
	7.	Problems	
	8.	Example: the Jump Caused by the Dirac $\delta$	

9. Satisfying the Initial Conditions	10. Integrating the Dirac $\delta$	
11. More Problems	12. Comments on Continuous and Discontinuous Functions	
F. Generalized Derivatives		288
1. Example	2. Example: Finding the Generalized Derivative of $y(t) = u(t - 1) \sin(t - 1)$	
3. Definition of the Generalized Derivative		
4. Examples	5. Properties	
6. Example: Checking the Generalized Solution of an Initial Value Problem	7. Example: Setting Up an Initial Value Problem	
8. Problems		
G. Analysis of Systems: the Transfer Function and the Convolution Integral		299
1. The "Systems Perspective." Input, Output, and the Quiescent State	2. The Transfer Function and the Transform Domain	
3. Examples	4. Sample Transfer-Function Problems with Answers	
5. Impulse Response and the Time Domain		
6. Convolution in Systems Analysis	7. Convolution as a Computational Technique in Laplace Transformation	
8. The Laplace Transform of the Convolution Integral	9. Examples	
10. Sample Problems with Answers	11. Problems	
12. Linearity Properties of the Convolution Integral		
13. Comments on the Analysis of Systems		
14. Example: Formula for the Solution of the First-Order Linear Equation with Constant Coefficients		
H. Further Properties of the Laplace Transform		342
1. The Initial Value Theorem and Final Value Theorem	2. Laplace Transforms of Periodic Functions and Series	
3. Sample Problems with Answers	4. Problems	
THE $J_\omega$ OPERATOR		355
I. Complex Arithmetic and the Complex Exponential		355
1. Complex Addition	2. The Complex Plane (Gaussian Plane or Argand Diagram)	
3. Complex Multiplication	4. The Complex Conjugate	
5. Division by Complex Numbers	6. The Complex Exponential	
7. Comparison of the Complex Exponential and the Real Exponential	8. Polar Coordinates for Complex Numbers	
9. Polar-Coordinate Representation Using the Complex Exponential	10. Finding Arguments from the Real and Imaginary Parts	
11. Sample Problems with Answers	12. Problems	
13. Complex Numbers as Vectors	14. The Roots of Quadratic Polynomials	

with Real Coefficients 15. The Partial-Fraction Decomposition with Complex Fractions 16. The Laplace Transform of the Complex Exponential 17. The Elementary Functions 18. More Problems

J. The $J_{\omega}$ Operator	377
1. Sinusoidal Functions 2. Problems 3. $J_{\omega}$ Operator Definition 4. Finding Steady-State Solutions 5. Sample Problems with Answers 6. Bode Plots 7. Examples 8. Problems	

## CHAPTER IV Linearity Explained and Used 399

A. Linear Operators, Familiar and Unfamiliar	400
1. The Familiar Linear Operators $\mathcal{L}$ and $J_{\omega}$ 2. More Examples of Linear Operators and Their Uses 3. A Description of Operators 4. Inverse Operators and the Solutions of Differential Equations 5. Sample Problems with Answers 6. Problems 7. The Operators $D^n$ 8. Linear Operators and Differential Equations with Constant Coefficients 9. Linear Differential Operators with Constant Coefficients 10. Sample Problems with Answers 11. Problems 12. Using Operators to Eliminate Variables: An Example 13. Elimination Method and Example 14. Sample Problems with Answers 15. Problems 16. Linear Differential Equations 17. Linearity, Nonlinearity, and Homogeneity 18. Sample Problems with Answers 19. Problems	

### LINEAR DIFFERENTIAL OPERATORS WITH CONSTANT COEFFICIENTS 434

B. The Characteristic Polynomial; Solutions of Homogeneous Equations	434
1. Example 2. The Characteristic Polynomial, Its Roots and Factors 3. The Rule for Real Solutions 4. Example: $p(s)$ Has Real, Equal Roots 5. Example: $p(s)$ Has Complex Roots 6. Example: $p(s)$ Has Real, Distinct Roots 7. More Examples of the Use of the Characteristic Polynomial 8. Remarks on the Practical Application of the Rule 9. Sample Problems with Answers 10. Problems 11. Determination of Damped and Undamped Sinusoidal Solutions 12. The Rule for Complex Solutions 13. Example and Remarks on the Practical Application of the Rule 14. Factoring Constant Coefficient Operators 15. Annihilation: Which Equations Have Given Solution?	

16. More Sample Problems with Answers	
17. Problems	
18. Last Words on Eliminating Variables	
19. Statement of the Refined Method	
20. Systems Containing More than Two Unknowns	
21. Sample Problems with Answers	
22. Problems	
C. The Method of Undetermined Coefficients for Solving Nonhomogeneous Equations	453
1. Example	
2. The Method	
3. Example	
4. Sample Problems with Answers	
5. Problems	
6. Simple Rule for the Results in the Simplest Case	
7. Examples of the Simplest Case	
8. Examples in Which the Simplest-Case Results Cannot Be Used	
9. Sample Problems with Answers	
10. Problems	
LINEAR DIFFERENTIAL OPERATORS WITH GENERAL COEFFICIENTS	460
D. The Equidimensional Equation	460
1. Examples	
2. Equidimensional Operators and Their Characteristic Polynomials	
3. The Rule for Homogeneous Equidimensional Equations	
4. Examples and Comments	
5. Sample Problems with Answers	
6. Problems	
7. Relation to Operators with Constant Coefficients	
E. Forms of a Homogeneous Solution; Linear Independence	468
1. Example: Rewriting the Solution of a Differential Equation	
2. Example: The Effect of Changing the Initial Point	
3. The Basic Initial System with Initial Point $t = t_0$	
4. More Examples of Basic Solution Systems	
5. Sample Problems with Answers	
6. Problems	
7. Linear Combinations and Linear Independence	
8. Techniques for Showing Linear Dependence or Independence	
9. Examples of the Use of the Linear-Independence Criterion	
10. Example of Reference to a Known Independent Collection	
11. Sample Problems with Answers	
12. Problems	
13. Remarks on the Rules for Using the Characteristic Polynomial	
14. Remarks: Linearly Independent Sets Are Minimal	
F. Variation of Parameters and Reduction of Order	490
1. Example and Method: A First-Order Equation	
2. Example: A Second-Order Equation	
3. Method of Variation of Parameters: The Second-Order Equation	
4. Method of Variation of Parameters: the Equation of Order $n$	
5. Example: An Equation of Order 3	
6. Reduction of Order	
7. Sample	

APPENDIX      Power-Series Solution Methods      505

1. Examples: Techniques of Power-Series Substitution
  2. Sample Problems with Answers
  3. Problems
  4. The Use of the Technique: Examples
  5. The Method of Frobenius
  6. Remarks on the Method of Frobenius and Its Use
  7. Sample Problems with Answers
  8. Problems
  9. Bessel Functions and Legendre Polynomials
  10. Sample Problem with Answer
- 

Answers to Selected Problems      524

Index      541