

Contents

Preface	vii
1 General Theory of Linear Programming	1
1.1 The Simplex and the Lexicographic Simplex Methods	1
1.2 The Duality Theorem	10
1.3 Farkas' Theorem on Linear Inequalities	13
1.4 von Neumann's Theorem on Two-Person Zero-Sum Games	15
1.5 The Dual and Lexicographic Dual Methods	17
1.6 Discussion of the Tableaux	20
1.7 Complementary Slackness Theorems	23
1.8 Exercises and Problems	27
2 Convex Polyhedra	35
2.1 Definitions	35
2.2 Parametric Representation of the Solutions of Homogeneous Linear Inequalities	42
2.3 Canonical Representation of Convex Polyhedra	48
2.4 Vertices of Convex Polyhedra	52
2.5 Exercises and Problems	54
3 Special Problems and Methods	59
3.1 Handling of Problems Containing Free Variables—Revised Methods	59
3.2 Individual Upper Bounds	62
3.3 The Parametric Method	67
3.4 Cutting Plane Sequences	71
3.5 Cutting Plane Method for the Solution of the All Integer Variable Linear Programming Problem	76

3.6	The Dantzig–Wolfe Decomposition Method	78
3.7	Miscellaneous Remarks	81
3.8	Exercises and Problems	83
4	Logconcave and Quasi-Concave Measures	87
4.1	Preliminary Notions	87
4.2	The Basic Theorems of Logconcave Measures	89
4.3	Logconvexity	95
4.4	Examples of Multivariate Logconcave and Logconvex Probability Densities	97
4.5	Inequalities for Sums and Integrals	99
4.6	Application to Probability Distributions: Generalization of Theorem 4.2.1	104
4.7	Logconcavity of Discrete Distributions	107
4.8	Theorems on the Binomial and Poisson Distributions	111
4.9	Exercises and Problems	121
5	Moment Problems	125
5.1	Introduction	125
5.2	Summary of the Chebyshev–Markov Theory and Related Results . .	128
5.3	Refined Lower and Upper Bounds for the Expectation of a Convex Function	137
5.4	General Moment Problems	139
5.5	Upper Bounds on the Expectation of a Multivariate Convex Function	146
5.6	Discrete Moment Problems	152
5.7	The Structure of the Dual Feasible Bases	158
5.8	Generalization and Solutions of Problems	163
5.9	Closed Form Bounds	168
5.9.1	Lower Bounds, μ_1, μ_2 are Given	172
5.9.2	Upper Bounds, μ_1, μ_2 are Given	173
5.9.3	Lower Bounds, μ_1, μ_2, μ_3 are Given	174
5.9.4	Upper Bounds, μ_1, μ_2, μ_3 are Given	176
5.10	Exercises and Problems	177
6	Bounding and Approximation of Probabilities	179
6.1	Introduction	179
6.2	Sharp Bounds on the Probability that at Least r out of n Events Occur, Given S_1, \dots, S_m	182
6.2.1	Lower Bounds, S_1, S_2 Given	186
6.2.2	Upper Bounds, S_1, S_2 Given	187
6.2.3	Lower Bounds, S_1, S_2, S_3 Given	187
6.2.4	Upper Bounds, S_1, S_2, S_3 Given	187
6.2.5	Upper Bounds, S_1, S_2, S_3, S_4 , Given	187
6.3	Hunter's Upper Bound	188

6.4	Application of Probability Bounds for the Solution of the Satisfiability Problem	190
6.5	Combined Use of Inclusion–Exclusion and Simulation to Estimate the Probability of a Composite Event	191
6.6	Approximation of Multivariate Normal, Gamma, and Dirichlet Probability Integrals	194
6.6.1	Multivariate Normal Distribution	195
6.6.2	A Multivariate Gamma Distribution	197
6.6.3	Dirichlet Distribution	200
6.6.4	Gradients	203
6.7	The Probability of a Rectangle in Case of a Multivariate Normal Distribution	208
6.8	A Hybrid Method to Compute Multivariate Normal Probabilities	210
6.9	Exercises and Problems	212
7	Statistical Decisions	219
7.1	Introduction	219
7.2	The Bernoulli Principle	221
7.3	Probability Maximization	223
7.4	Ensuring Safety through Utility	225
7.5	Choosing Efficient Points	225
7.6	The Neyman–Pearson Lemma	226
7.7	Bayesian Decisions	227
7.8	Decision when the Probability Distribution is Unknown (The Minimax Principle)	228
7.9	The Sequential Probability Ratio Test	229
7.10	Two-Stage Methods	229
7.11	Wald’s Theory of Statistical Decision Function	231
8	Static Stochastic Programming Models	233
8.1	Introduction	233
8.2	Probability Maximization	235
8.3	Programming under Probabilistic Constraints	235
8.4	Constraints Involving Conditional Expectations and Related Measures	239
8.5	Handling a Random Objective Function	243
8.6	Models where Infeasibility is Penalized	247
8.7	The Newsboy Problem	252
8.8	Simultaneous Use of Penalties and Probabilistic Constraint	253
8.9	Utility Functions and Deterministic Equivalents	255
8.10	Stochastic Programming with Multiple Objective Functions	257
8.11	Game Theoretical Formulation	259
8.12	Exercises and Problems	264

9 Solutions of the Simple Recourse Problem	269
9.1 Introduction	269
9.2 Primal Method for the Solution of the Simple Recourse Problem	271
9.3 Dual Method for the Solution of the Simple Recourse Problem	278
9.4 Applications for Deterministic Problems	289
9.5 The Case of the Continuous Distribution	291
9.6 Allocation of Aircraft to Routes under Uncertain Demand	294
9.7 Exercises and Problems	297
10 Convexity Theory of Probabilistic Constrained Problems	301
10.1 Introduction	301
10.2 General Convexity Statements	302
10.3 Some Concavity and Quasi-Concavity Theorems for Probability Distribution Functions	306
10.4 Convexity Statements for Random Linear Constraints	311
10.5 Exercises and Problems	316
11 Programming under Probabilistic Constraint and Maximizing Probabilities under Constraints	319
11.1 Introduction	319
11.2 The Use of the SUMT Interior Point Method with Logarithmic Barrier Functions	320
11.3 Application to a Reliability Type Inventory Model	326
11.4 Application to Serially Linked Reservoir System Design	331
11.5 The Use of a Supporting Hyperplane Method	337
11.6 Numerical Examples	340
11.7 Application of the GRG Method	346
11.8 Solution by a Primal-Dual Algorithm	349
11.9 Probabilistic Constraints Involving Discrete Distribution	351
11.10 Applications in Statistics	357
11.11 A Wafer Design Problem in Semiconductor Manufacturing	363
11.12 The Use of Probability Bounding Techniques in Probabilistic Constrained Stochastic Programming	365
11.13 Exercises and Problems	367
12 Two-Stage Stochastic Programming Problems	373
12.1 Formulation of the Problem	373
12.2 Mathematical Properties of the Recourse Problem	377
12.3 Solution of the Recourse Problem by Basis Decomposition Technique when ξ has a Discrete Distribution	380
12.4 Solution of the Recourse Problem by the L-Shaped Method	389
12.5 Solution of the General Recourse Problem by Discretization	395
12.6 Sublinear Upper Bounding Technique for the Recourse Function	400
12.7 Regularized Decomposition Method for Minimizing a Sum of Polyhedral Functions	403

12.8	Stochastic Decomposition and Conditional Stochastic Decomposition	406
12.9	Stochastic Quasigradient Methods	414
12.10	Two-Stage Stochastic Programming Formulations Using Probabilistic Constraint	417
12.11	Two-Stage Stochastic Integer Programming	420
12.12	Exercises and Problems	421
13	Multi-Stage Stochastic Programming Problems	425
13.1	Formulation of the Problem	425
13.2	Probabilistic Constrained Formulation	431
13.3	Basis Decomposition Technique Applied to a Multi-Stage Stochastic Programming Problem	434
13.4	L-Shaped Technique Applied to a Multi-Stage Stochastic Programming Problem	439
13.5	The Method of Scenario Aggregation	444
14	Special Cases and Selected Applications	447
14.1	A Network Recourse Problem	447
14.2	Electric Power Generation Capacity Expansion under Uncertainty	448
14.3	Models Including the Transmission System	452
14.4	Computing Power System Reliability	458
14.5	Optimal Scheduling of a Hydrothermal Generating System	466
14.6	Optimal Control of a Storage Level	468
14.7	An Example for Optimal Control of Reservoirs	479
14.8	Two-Sector Multi-Stage Economic Planning	485
14.9	A PERT Optimization Problem	486
14.10	Finance Problems	492
14.11	Diet and Animal Feed Problems	499
15	Distribution Problems	501
15.1	Formulation of the Problem	501
15.2	The Random Linear Programming Problem	502
15.3	The Continuity of the Optimum Value of a Linear Programming Problem	507
15.4	Computation of Characteristics of the Random Optimum Value	509
15.5	Asymptotic Distribution of the Optimum in Case of a Highly Stable Basis	513
15.6	Laws of Large Numbers for Random Linear Programs	519
15.7	Laws of Large Numbers for Random Knapsack Problems	526
15.8	The Beardwood–Halton–Hammersley Theorem for the Random Traveling Salesman Problem	533
15.9	Some Inequalities	533
15.10	Exercises and Problems	537

Appendix. The Multivariate Normal Distribution	541
Bibliography	551
Author Index	589
Subject Index	595