

# Contents

<b>1</b>	<b>ORTHOGONAL FUNCTIONS</b>	<b>1</b>
1.1	Introduction, 1	
1.2	Preliminary Definitions, 4	
1.3	Generalized Fourier Series, 6	
1.4	Bessel's Inequality, 7	
1.5	Parseval's Equation, 7	
1.6	An Orthogonalization Process, 9	
1.7	An Example, 11	
1.8	Generating Functions and Recurrence Relations, 13	
1.9	Sturm-Liouville Systems, 17	
1.10	Orthogonal Functions of Two Variables, 20	
1.11	Rodrigues' Formula, 22	
<b>2</b>	<b>FOURIER SERIES</b>	<b>25</b>
2.1	Fourier Trigonometric Series, 25	
2.2	An Example, 27	
2.3	Even and Odd Functions, 29	
2.4	Expansions of Even and Odd Functions; Half-Range Series, 30	
2.5	Operations with Fourier Series, 34	
2.6	Parseval's Theorem, 38	
2.7	Fourier Integral, 39	
<b>3</b>	<b>SERIES SOLUTION OF DIFFERENTIAL EQUATIONS</b>	<b>42</b>
3.1	Preliminary Example, 42	
3.2	Generalized Series Solution, 45	
3.3	Existence of the Series Solution, 46	
3.4	A Second Solution, 48	
<b>4</b>	<b>LEGENDRE FUNCTIONS</b>	<b>51</b>
4.1	Legendre Polynomials, 51	
4.2	Legendre Functions of the Second Kind, 54	

4.3	Generating Function for $P_n(x)$ , 55	
4.4	Rodrigues' Formula, 57	
4.5	Orthogonality of the $P_n(x)$ , 58	
4.6	Recurrence Relations for $P_n(x)$ , 59	
4.7	Series Expansions Involving $P_n(x)$ , 61	
4.8	Associated Legendre Functions, 64	
4.9	Recurrence Relations for $P_n^m(x)$ , 65	
4.10	Orthogonality and Generating Function of $P_n^m(x)$ , 67	
4.11	Spherical Harmonics, 70	
4.12	Series Expansions Involving $P_n^m(x)$ , 72	
4.13	Another Expression for $Q_n(x)$ , 74	
4.14	Recurrence Relations for $Q_n(x)$ , 75	
4.15	Generating Function for $Q_n(x)$ , 77	
<b>5</b>	<b>THE GAMMA FUNCTION</b>	<b>79</b>
5.1	Integral Definition, 79	
5.2	Euler's Constant, 81	
5.3	Weierstrass' Definition, 83	
5.4	Other Forms for the Gamma Function, 84	
5.5	Logarithmic Derivative, 86	
<b>6</b>	<b>BESSEL FUNCTIONS</b>	<b>88</b>
6.1	Bessel's Differential Equation, 88	
6.2	Bessel Function of the Second Kind, 91	
6.3	Generating Function for $J_n(x)$ , 94	
6.4	Recurrence Relations, 97	
6.5	Spherical Bessel Functions, 99	
6.6	Zeros of $J_n(x)$ , 101	
6.7	Orthogonality of $J_n(x)$ , 102	
6.8	Integral Relations, 105	
6.9	Some Properties of $Y_n(x)$ , 109	
6.10	An Orthogonality Relation Involving $Y_n(x)$ , 111	
<b>7</b>	<b>BOUNDARY-VALUE PROBLEMS</b>	<b>113</b>
7.1	Linear Operators and Boundary-Value Problems, 113	
7.2	Principle of Superposition, 115	
7.3	Infinite Series of Solutions, 116	
7.4	Separation-of-Variables Method, 119	
7.5	Summary of the Method, 120	
7.6	An Example, 122	
7.7	Limitations of the Method, 125	
<b>8</b>	<b>PARTIAL DIFFERENTIAL EQUATIONS OF MATHEMATICAL PHYSICS</b>	<b>129</b>
8.1	Helmholtz Equation, 129	
8.2	Wave Equation, 131	

8.3	Vibrating String, 131	
8.4	Vibrating Membrane, 134	
8.5	Diffusion Equation, 136	
8.6	Laplace's Equation, 140	
<b>9</b>	<b>HERMITE POLYNOMIALS</b>	<b>145</b>
9.1	Definition, 145	
9.2	Generating Function, 146	
9.3	Recurrence Relations, 147	
9.4	Orthogonality, 148	
9.5	Expansion of Functions in Terms of $H_n(x)$ , 149	
9.6	General Solution of Hermite's Equation, 151	
9.7	Hermite's Orthogonal Functions, 152	
<b>10</b>	<b>LAGUERRE POLYNOMIALS</b>	<b>154</b>
10.1	Definition, 154	
10.2	Recurrence Relations and Differential Equation, 155	
10.3	Rodrigues' Formula, 158	
10.4	Orthogonality, 158	
10.5	Simple Laguerre Polynomials $L_n(x)$ , 160	
10.6	Example from Quantum Mechanics, 163	
<b>11</b>	<b>CHEBYSHEV POLYNOMIALS</b>	<b>166</b>
11.1	Definitions, 166	
11.2	Recurrence Relations and Differential Equations, 167	
11.3	Orthogonality Relations, 169	
11.4	Generating Functions, 171	
11.5	Rodrigues' Formula, 173	
11.6	Zeros of $T_n(x)$ and Associated Properties, 174	
11.7	Expansions in Series of Chebyshev Polynomials, 175	
11.8	An Approximation Example, 179	
11.9	Boundary-Value Problems, 180	
<b>12</b>	<b>MATHIEU FUNCTIONS</b>	<b>183</b>
12.1	Mathieu's Equation, 183	
12.2	Properties of Elliptic-Cylinder Coordinates, 184	
12.3	Solution of Mathieu's Equation, 185	
12.4	Nature of the General Solutions, 189	
12.5	Orthogonality of the Periodic Solutions, 191	
12.6	An Example, 193	
<b>13</b>	<b>OTHER SPECIAL FUNCTIONS</b>	<b>196</b>
13.1	Hypergeometric Function, 196	
13.2	Jacobi Polynomials, 200	
13.3	Rodrigues' Formula for Jacobi Polynomials, 201	
13.4	Orthogonality of the Jacobi Polynomials, 202	
13.5	Bessel Polynomials, 205	
13.6	Some Related Polynomials, 206	

<b>14 LAPLACE AND FOURIER TRANSFORMS</b>	<b>210</b>
14.1 Introduction, 210	
14.2 Laplace Transform, 211	
14.3 Solutions of Differential Equations, 214	
14.4 Convolution, 216	
14.5 Fourier Transform, 219	
14.6 Properties of the Fourier Transform, 221	
14.7 System Functions, 223	
14.8 Filter Theory, 224	
<b>15 STURM-LIOUVILLE TRANSFORMS</b>	<b>229</b>
15.1 Definition, 229	
15.2 Finite Fourier Sine and Cosine Transforms, 230	
15.3 Hankel Transform, 234	
15.4 Legendre Transform, 236	
15.5 Laguerre Transform, 238	
15.6 Hermite Transform, 239	
15.7 Other Transforms, 239	
<b>16 A GENERAL CLASS OF ORTHOGONAL POLYNOMIALS</b>	<b>242</b>
16.1 A Unifying Concept, 242	
16.2 Orthogonality of $G_n$ , 244	
16.3 Norm of $G_n(-1, \alpha + \beta, -\alpha\beta, h, k, C_n, x)$ , 245	
16.4 Infinite Intervals, 248	
16.5 Generating Functions, 249	
16.6 Summary, 252	
<b>APPENDIX</b>	
<b>A PROPERTIES OF INFINITE SERIES</b>	<b>255</b>
A.1 Convergent Series, 255	
A.2 Uniformly Convergent Series, 256	
A.3 Power Series, 258	
<b>B CONVERGENCE OF THE FOURIER SERIES</b>	<b>259</b>
B.1 Sufficiency for Convergence, 259	
<b>C TABLES</b>	<b>262</b>
1 Laplace Transforms, 262	
2 Finite Sine Transforms, 264	
3 Finite Cosine Transforms, 264	
4 Summary of Properties of Polynomial Sets $\{\phi_n(x)\}$ , 265	
5 Generating Functions, 266	
<b>BIBLIOGRAPHY</b>	<b>267</b>
<b>INDEX</b>	<b>269</b>