

Contents

<i>Preface</i>	xi
<i>Acknowledgments</i>	xiii

Chapter 1 Introduction

1.1 Structure of Sequential Decision Models	1
1.2 Discrete-Time Stochastic Optimal Control Problems—Measurability Questions	5
1.3 The Present Work Related to the Literature	13

Part I ANALYSIS OF DYNAMIC PROGRAMMING MODELS

Chapter 2 Monotone Mappings Underlying Dynamic Programming Models

2.1 Notation and Assumptions	25
2.2 Problem Formulation	28
2.3 Application to Specific Models	29
2.3.1 Deterministic Optimal Control	30
2.3.2 Stochastic Optimal Control—Countable Disturbance Space	31
2.3.3 Stochastic Optimal Control—Outer Integral Formulation	35
2.3.4 Stochastic Optimal Control—Multiplicative Cost Functional	37
2.3.5 Minimax Control	38

Chapter 3 Finite Horizon Models

3.1 General Remarks and Assumptions	39
3.2 Main Results	40
3.3 Application to Specific Models	47

<i>Chapter 4</i>	Infinite Horizon Models under a Contraction Assumption	
4.1	General Remarks and Assumptions	52
4.2	Convergence and Existence Results	53
4.3	Computational Methods	58
4.3.1	Successive Approximation	59
4.3.2	Policy Iteration	63
4.3.3	Mathematical Programming	67
4.4	Application to Specific Models	68
<i>Chapter 5</i>	Infinite Horizon Models under Monotonicity Assumptions	
5.1	General Remarks and Assumptions	70
5.2	The Optimality Equation	71
5.3	Characterization of Optimal Policies	78
5.4	Convergence of the Dynamic Programming Algorithm—Existence of Stationary Optimal Policies	80
5.5	Application to Specific Models	88
<i>Chapter 6</i>	A Generalized Abstract Dynamic Programming Model	
6.1	General Remarks and Assumptions	92
6.2	Analysis of Finite Horizon Models	94
6.3	Analysis of Infinite Horizon Models under a Contraction Assumption	96
<i>Part II</i> STOCHASTIC OPTIMAL CONTROL THEORY		
<i>Chapter 7</i>	Borel Spaces and Their Probability Measures	
7.1	Notation	102
7.2	Metrizable Spaces	104
7.3	Borel Spaces	117
7.4	Probability Measures on Borel Spaces	122
7.4.1	Characterization of Probability Measures	122
7.4.2	The Weak Topology	124
7.4.3	Stochastic Kernels	134
7.4.4	Integration	139
7.5	Semicontinuous Functions and Borel-Measurable Selection	145
7.6	Analytic Sets	156
7.6.1	Equivalent Definitions of Analytic Sets	156
7.6.2	Measurability Properties of Analytic Sets	166
7.6.3	An Analytic Set of Probability Measures	169
7.7	Lower Semianalytic Functions and Universally Measurable Selection	171
<i>Chapter 8</i>	The Finite Horizon Borel Model	
8.1	The Model	188

8.2	The Dynamic Programming Algorithm—Existence of Optimal and ϵ -Optimal Policies	194
8.3	The Semicontinuous Models	208
Chapter 9 The Infinite Horizon Borel Models		
9.1	The Stochastic Model	213
9.2	The Deterministic Model	216
9.3	Relations between the Models	218
9.4	The Optimality Equation—Characterization of Optimal Policies	225
9.5	Convergence of the Dynamic Programming Algorithm—Existence of Stationary Optimal Policies	229
9.6	Existence of ϵ -Optimal Policies	237
Chapter 10 The Imperfect State Information Model		
10.1	Reduction of the Nonstationary Model—State Augmentation	242
10.2	Reduction of the Imperfect State Information Model—Sufficient Statistics	246
10.3	Existence of Statistics Sufficient for Control	259
10.3.1	Filtering and the Conditional Distributions of the States	260
10.3.2	The Identity Mappings	264
Chapter 11 Miscellaneous		
11.1	Limit-Measurable Policies	266
11.2	Analytically Measurable Policies	269
11.3	Models with Multiplicative Cost	271
Appendix A	The Outer Integral	273
Appendix B	Additional Measurability Properties of Borel Spaces	
B.1	Proof of Proposition 7.35(e)	282
B.2	Proof of Proposition 7.16	285
B.3	An Analytic Set Which Is Not Borel-Measurable	290
B.4	The Limit σ -Algebra	292
B.5	Set Theoretic Aspects of Borel Spaces	301
Appendix C	The Hausdorff Metric and the Exponential Topology	
References		312
<i>Table of Propositions, Lemmas, Definitions, and Assumptions</i>		317
<i>Index</i>		321