Contents

Preface: ix

Chapter 1: Typical Elliptic Problems 1

1. Introduction 1
2. Dirichlet and related problems 3
3. Membrane; source problems 5
4. Two-endpoint problems 6
5. Helmholtz equation 10
6. Potential flow problems 12
7. Problems from elasticity 14
8. Multigroup diffusion 18
9. Some nonlinear problems 19

Chapter 2: Classical Analysis 21

1. Separation of variables 21
2. Complex variable techniques 25
3. Integral formulas 27
4. Green's identities 30
5. Inner products; norms 34
6. Variational principles 38
7. Maximum principle 41
8. Smoothness 42

Chapter 3: Difference Approximations 45

1. Arithmetizing analysis 45
2. Poisson equation; ELLPACK 48
3. Discretizing boundary conditions 52
4. Linear source problem 57
5. Band elimination 61
6. Error estimation 68
7. Orders of accuracy 73
8. Effects of smoothness 78
9. Error bounds 84
10. Higher-order accuracy 87
11. The HODIE method 91

Chapter 4: Direct and Iterative Methods 97

1. Direct methods 97
2. Profile elimination 100
3. Graph of a matrix; nested dissection 102
4. Asymptotic cost estimates 107
5. Marching methods 110
6. Tensor product solvers 112
7. Iterative methods 118
8. Other one-step iterative methods 121
9. Physical analogies; relaxation methods 124
10. Convergence criteria 127
11. Estimating $\rho(G)$ 130
12. Property A 132
13. Point SOR 136
14. Property Y; numerical examples 141
15. Frankel's method 144

Chapter 5: Accelerating Convergence 149

1. Introduction 149
2. Improvements in SOR 151
3. Conjugate gradient methods 155
4. The CSI and CCSI methods 158
5. Chebyshev acceleration 162
6. Conjugate gradient acceleration 166
7. Adaptive parameter selection; stopping criteria 170
8. ADI methods 173
9. Stationary ADI 176
10. Multigrid; mesh refinement 178

Chapter 6: Direct Variational Methods 183

1. Introduction 183
2. Rayleigh-Ritz method 186
3. Approximating subspaces 188
4. Proving convergence 194
5. Galerkin methods 197
6. Collocation 199
7. Eigenproblems 202
8. Domain constants 206
9. Nonlinear elliptic problems 208
10. Newton's method 210
11. Steepest descent 213
12. Davidon's method; fixpoint methods 217

Chapter 7: Finite Element Approximations 221

1. Introduction 221
2. Univariate interpolation 223
3. Best approximation 225
4. Typical finite elements 228
5. Hermite interpolants; univariate splines 231
6. Triangular elements 235
7. Tensor product approximations 238
8. Rectangular polygons 239
9. Blending 242
10. Boundary elements 246

Chapter 8: Integral Equation Methods 251

1. Introduction 251
2. Green's function; Peano-Sard kernels 253
3. Least squares method 258
4. Boundary element methods 262
5. Relation to conformal mapping 264
6. Conformal mapping 266
7. Theodorsen's method 270
8. Three-dimensional problems 272
9. Capacitance matrix method 273
10. Helmholtz equation 275
11. Rigorizing potential theory 276
12. Méthode de balayage 279

Chapter 9: ELLPACK 283

1. Overview 283
2. Processing of an ELLPACK program 285
3. Problem definition 288
4. DISCRETIZATION modules 291
5. INDEXING and SOLUTION modules 292
6. OUTPUT, OPTIONS, and PROCEDURE modules 293
7. Fortran segments 294
9. TRIPLE segments 295
9. Sample ELLPACK programs 297
10. Additional capabilities 301

Bibliography: 303

Index: 315