

Contents

Foreword	ix
Preface	xi
Chapter 1. Introduction	1
1.1 Introduction	1
1.2 Linear models of hyperbolic equations	2
1.3 Approximation	4
1.4 Finite element Galerkin semi-discretizations	5
1.5 Toeplitz operators	7
1.6 B -spline Galerkin semi-discretizations	8
1.7 Fully discrete approximations	11
1.8 Fourier analysis	12
1.9 \mathcal{L}_2 -norms and Parseval's equality	12
1.10 Relationship between continuous and discrete transforms	13
1.11 Fourier analysis of numerical approximations	16
1.12 Sampling of the initial data	16
Chapter 2. Fourier Analysis of the Accuracy of Semi-Discretizations	19
2.1 Introduction	19
2.2 Sinusoidal trial solutions	19
2.3 Fourier transforms and global errors	23
2.4 Relation to classical truncation error analysis	24
2.5 Finite element Galerkin semi-discretizations	26
2.6 A generalization	27
2.7 Semi-discretization of a conservation law	29
2.8 Time frequency	30
2.9 The wave equation	31
2.10 Implicit semi-discretizations of the wave equation	33
Chapter 3. Higher Order Semi-Discretizations	35
3.1 Synthesis in the frequency domain	35
3.2 Velocity error	37
3.3 Limiting case ($K \rightarrow \infty$)	37
3.4 Relation to the cardinal function	37

3.5	<i>B</i> -spline Galerkin semi-discretizations	40
3.6	An equivalence of bases	40
3.7	Equivalence with collocation	41
3.8	Fourier analysis of the algorithms obtained with <i>B</i> -splines .	43
3.9	Convergence rates	43
3.10	<i>B</i> -spline semi-discretizations of the wave equation . . .	46
3.11	Analysis	48
Chapter 4. Full Discretizations		51
4.1	Fourier analysis	51
4.2	Stability	54
4.3	Velocity and amplitude error	58
4.4	Examples	59
4.5	Time marching methods for second order equations . . .	61
Chapter 5. Damping, Diffusion and Filtering		63
5.1	Spurious diffusion	63
5.2	Limitations of the spurious diffusion model	65
5.3	Spurious diffusion in discretizations of the wave equation .	66
5.4	Filtering	69
5.5	Low pass filters	70
5.6	Flat low pass filters	70
5.7	Fast Fourier transform filtering	71
5.8	Effect of filtering on damping and numerical stability . . .	74
Chapter 6. Group Velocity		75
6.1	Introduction	75
6.2	Group velocity and energy propagation	77
6.3	Group velocity of the simple 3-point finite differences semi- discretization	77
6.4	Group velocity of other 3-point semi-discretizations . . .	78
6.5	Wave analysis	80
6.6	Wave analysis of other semi-discretizations	82
Chapter 7. Time-Fourier Transforms		85
7.1	Introduction	85
7.2	Numerical phase velocity and wavelength	87
7.3	Relation to <i>x</i> -Fourier transforms	90
7.4	Cut-off frequency	92
7.5	Energy	95
7.6	Group velocity of the two fundamental types of solutions .	95

7.7	Reflection at a downstream boundary	97
7.8	Convergence rates	99
7.9	A 3-point boundary formula	101
Chapter 8. Fourier Analysis and \mathcal{L}_2 -Norm of the Global Error		103
8.1	Introduction	103
8.2	Examples	104
8.3	Relation with convergence rates analysis	107
8.4	Asymptotic approximations	107
Chapter 9. Spectral Methods		109
9.1	Introduction	109
9.2	Truncated Fourier series method	110
9.3	Analysis	111
9.4	Fourier “collocation” method	112
Chapter 10. Equations in Two Dimensions: Anisotropy		115
10.1	The advection equation in two dimensions	115
10.2	Anisotropy of the approximation on a square grid	116
10.3	Analysis of other explicit formulae	119
10.4	Implicit approximations	120
10.5	The wave equation in the plane	122
10.6	9-point semi-discretizations of the wave equation	128
Bibliography		131
Index		137