

Contents

Introduction	1
I Mathematical Preliminaries	3
1 Conformal Transformations and Conformal Killing Fields	3
1.1 Semi-Riemannian Manifolds	3
1.2 Conformal Transformations	5
1.3 Conformal Killing Fields	9
1.4 Classification of Conformal Transformations	12
1.4.1 Case 1: $n = p + q > 2$	12
1.4.2 Case 2: Euclidean Plane ($p = 2, q = 0$) . . .	16
1.4.3 Case 3: Minkowski Plane ($p = q = 1$)	18
2 The Conformal Group	20
2.1 Conformal Compactification of $\mathbb{R}^{p,q}$	20
2.2 The Conformal Group of $\mathbb{R}^{p,q}$ for $p + q > 2$	24
2.3 The Conformal Group of $\mathbb{R}^{2,0}$	28
2.4 The Conformal Group of $\mathbb{R}^{1,1}$	32
3 Central Extensions of Groups	36
3.1 Central Extensions	36
3.2 Quantization of Symmetries	39
3.3 Equivalence of Central Extensions	44
4 Central Extensions of Lie Algebras and Bargmann's Theorem	47
4.1 Central Extensions and Equivalence	47
4.2 Bargmann's Theorem	51
5 The Virasoro Algebra	56
5.1 Witt Algebra and Infinitesimal Conformal Transformations of the Minkowski Plane	56
5.2 Witt Algebra and Infinitesimal Conformal Transformations of the Euclidean Plane	58

5.3	The Virasoro Algebra as a Central Extension of the Witt Algebra	60
II First Steps Towards Conformal Field Theory		65
6	Representation Theory of the Virasoro Algebra	65
6.1	Unitary and Highest-Weight Representations	65
6.2	Verma Modules	66
6.3	The Kac Determinant	69
6.4	Indecomposability and Irreducibility of Representations	74
7	Projective Representations of $\text{Diff}_+(\mathbb{S})$ and More	76
8	String Theory as a Conformal Field Theory	78
8.1	Action Functionals and Equations of Motion for Strings	78
8.2	Quantization	87
9	Foundations of Two-Dimensional Conformal Quantum Field Theory	95
9.1	Axioms for Two-Dimensional Euclidean Quantum Field Theory	95
9.2	Conformal Fields and the Energy-Momentum Tensor	101
9.3	Primary Fields, Operator Product Expansion and Fusion	104
9.4	Other Approaches to Axiomatization	108
10	Mathematical Aspects of the Verlinde Formula	110
10.1	The Moduli Space of Representations and Theta Functions	110
10.2	The Verlinde Formula	118
10.3	Fusion Rules for Surfaces with Marked Points	121
10.4	Combinatorics on Fusion Rings	129
References		132
Index		138