

Contents

Preface and Introduction	xiii
1 Introduction: Applications and Issues	1
1.0 Outline of Chapter	1
1.1 The Robbins–Monro Algorithm	2
1.1.1 Introduction	2
1.1.2 Finding the Zeros of an Unknown Function	5
1.1.3 A Linear Pattern Classifier: Best Linear Least Squares Fit	8
1.1.4 Minimization by Recursive Monte Carlo	11
1.2 The Kiefer–Wolfowitz Procedure	13
1.2.1 The Basic Procedure	13
1.2.2 Random Directions	16
1.3 Extensions of the Algorithms: Variance Reduction, Robustness, Iterate Averaging, Constraints, and Convex Optimization	18
1.3.1 A Variance Reduction Method	18
1.3.2 Constraints	20
1.3.3 Averaging of the Iterates: “Polyak Averaging”	21
1.3.4 Robust Algorithms	22
1.3.5 Nonexistence of the Derivative at Some θ	22
2 Applications to Learning, State Dependent Noise, and Queueing	25
2.0 Outline of Chapter	25

2.1	An Animal Learning Model	26
2.2	A Neural Network	29
2.3	Q -Learning	32
2.4	State Dependent Noise: A Motivational Example	35
2.5	Optimization of a GI/G/1 Queue	38
2.5.1	Derivative Estimation and Infinitesimal Perturbation Analysis: A Brief Review	39
2.5.2	The Derivative Estimate for the Queueing Problem	41
2.6	Passive Stochastic Approximation	45
3	Applications in Signal Processing and Adaptive Control	47
3.0	Outline of Chapter	47
3.1	Parameter Identification and Tracking	48
3.1.1	The Classical Model	48
3.1.2	ARMA and ARMAX Models	51
3.2	Tracking Time Varying Systems: An Adaptive Step Size Algorithm	53
3.2.1	The Algorithm	53
3.2.2	Some Data	56
3.3	Feedback and Averaging in the Identification Algorithm	58
3.4	Applications in Communication Theory	60
3.4.1	Adaptive Noise Cancellation and Disturbance Rejection	60
3.4.2	Adaptive Equalizers	62
4	Mathematical Background	67
4.0	Outline of Chapter	67
4.1	Martingales, Submartingales, and Inequalities	68
4.2	Ordinary Differential Equations	72
4.2.1	Limits of a Sequence of Continuous Functions	72
4.2.2	Stability of Ordinary Differential Equations	74
4.3	Projected ODE	77
4.4	Stochastic Stability and Perturbed Stochastic Liapunov Functions	80
5	Convergence with Probability One: Martingale Difference Noise	85
5.0	Outline of Chapter	85
5.1	Truncated Algorithms: Introduction	87
5.2	The ODE Method: A Basic Convergence Theorem	93
5.2.1	Assumptions and the Main Convergence Theorem	93
5.2.2	Chain Recurrence	102
5.3	A General Compactness Method	107
5.3.1	The Basic Convergence Theorem	107

5.3.2	Sufficient Conditions for the Rate of Change Condition	109
5.3.3	The Kiefer–Wolfowitz Algorithm	113
5.4	Stability and Stability–ODE Methods	114
5.5	Soft Constraints	120
5.6	Random Directions, Subgradients, and Differential Inclusions	122
5.7	Convergence for the Lizard Learning and Pattern Classification Problems	125
5.7.1	The Lizard Learning Problem	125
5.7.2	The Pattern Classification Problem	126
5.8	Convergence to a Local Minimum: A Perturbation Method	127
6	Convergence with Probability One: Correlated Noise	135
6.0	Outline of Chapter	135
6.1	A General Compactness Method	136
6.1.1	Introduction and General Assumptions	136
6.1.2	The Basic Convergence Theorem	140
6.1.3	Local Convergence Results	143
6.2	Sufficient Conditions for the Rate of Change Assumptions: Laws of Large Numbers	144
6.3	Perturbed State Criteria for the Rate of Change Assumptions	146
6.3.1	Introduction to Perturbed Test Functions	146
6.3.2	General Conditions for the Asymptotic Rate of Change	148
6.3.3	Alternative Perturbations	151
6.4	Examples Using State Perturbation	154
6.5	Kiefer–Wolfowitz Algorithms	157
6.6	A State Perturbation Method and State Dependent Noise	159
6.7	Stability Methods	162
6.8	Differential Inclusions and the Parameter Identification Problem	167
6.9	State Perturbation–Large Deviations Methods	168
6.10	Large Deviations Estimates	173
6.10.1	Two-Sided Estimates	173
6.10.2	Upper Bounds and Weaker Conditions	179
6.10.3	Escape Times	182
7	Weak Convergence: Introduction	185
7.0	Outline of Chapter	185
7.1	Introduction	186
7.2	Martingale Difference Noise	189

7.3	Weak Convergence	198
7.3.1	Definitions	198
7.3.2	Basic Convergence Theorems	201
7.4	Martingale Limit Processes and the Wiener Process	205
7.4.1	Verifying that a Process Is a Martingale	205
7.4.2	The Wiener Process	207
7.4.3	A Perturbed Test Function Method for Verifying Tightness and the Wiener Process	208
8	Weak Convergence Methods for General Algorithms	213
8.0	Outline of Chapter	213
8.1	Assumptions: Exogenous Noise and Constant Step Size	215
8.2	Convergence: Exogenous Noise	218
8.2.1	Constant Step Size: Martingale Difference Noise	218
8.2.2	Correlated Noise	225
8.2.3	Step Size $\epsilon_n \rightarrow 0$	228
8.2.4	Random ϵ_n	231
8.2.5	Differential Inclusions	231
8.3	The Kiefer–Wolfowitz Algorithm	232
8.3.1	Martingale Difference Noise	232
8.3.2	Correlated Noise	234
8.4	Markov State Dependent Noise	238
8.4.1	Constant Step Size	238
8.4.2	Decreasing Step Size $\epsilon_n \rightarrow 0$	242
8.4.3	The Invariant Measure Method: Constant Step Size	244
8.4.4	An Alternative Form	246
8.5	Unconstrained Algorithms	247
9	Applications: Proofs of Convergence	251
9.0	Outline of Chapter	251
9.1	Average Cost per Unit Time Criteria: Introduction	252
9.1.1	General Comments	252
9.1.2	A Simple Illustrative SDE Example	254
9.2	A Continuous Time Stochastic Differential Equation Example	258
9.3	A Discrete Example: A GI/G/1 Queue	263
9.4	Signal Processing Problems	266
10	Rate of Convergence	273
10.0	Outline of Chapter	273
10.1	Exogenous Noise: Constant Step Size	274
10.1.1	Martingale Difference Noise	275
10.1.2	Correlated Noise	283
10.2	Exogenous Noise: Decreasing Step Size	286

10.3	The Kiefer–Wolfowitz Algorithm	290
10.3.1	Martingale Difference Noise	290
10.3.2	Correlated Noise	294
10.4	Tightness of the Normalized Iterates: Decreasing Step Size, W.P.1 Convergence	298
10.4.1	Martingale Difference Noise: Robbins–Monro Algorithm	298
10.4.2	Correlated Noise	301
10.4.3	The Kiefer–Wolfowitz Algorithm	303
10.5	Tightness of the Normalized Iterates: Weak Convergence .	305
10.5.1	The Unconstrained Algorithm	305
10.5.2	The Constrained Algorithm and Local Methods . .	307
10.6	Weak Convergence to a Wiener Process	310
10.7	Random Directions: Martingale Difference Noise	315
10.7.1	Comparison of Algorithms	317
10.8	State Dependent Noise	321
11	Averaging of the Iterates	327
11.0	Outline of Chapter	327
11.1	Rate of Convergence of the Averaged Iterates: Minimal Window of Averaging	330
11.1.1	The Robbins–Monro Algorithm: Decreasing Step Size	330
11.1.2	Constant Step Size	333
11.1.3	The Kiefer–Wolfowitz Algorithm	333
11.2	A Two Time Scale Interpretation	335
11.3	Maximal Window of Averaging	336
11.4	The Parameter Identification Problem: An Optimal Algorithm	344
12	Distributed/Decentralized and Asynchronous Algorithms	347
12.0	Outline of Chapter	347
12.1	Examples	349
12.1.1	Introductory Comments	349
12.1.2	Pipelined Computations	350
12.1.3	A Distributed and Decentralized Network Model .	352
12.1.4	Multiaccess Communications	354
12.2	Introduction: Real-Time Scale	355
12.3	The Basic Algorithms	360
12.3.1	Constant Step Size: Introduction	360
12.3.2	Martingale Difference Noise	362
12.3.3	Correlated Noise	368
12.3.4	Infinite Time Analysis	370
12.4	Decreasing Step Size	373
12.5	State Dependent Noise	378

12.6	Rate of Convergence: The Limit Rate Equations	381
12.7	Stability and Tightness of the Normalized Iterates	386
12.7.1	The Unconstrained Algorithm	386
12.8	Convergence for Q -Learning: Discounted Cost	390
	References	393
	Symbol Index	409
	Index	413