

CONTENTS

PREFACE	v
NOTE TO THE READER	viii
CHAPTER 1 INTRODUCTION	1
1.1 Science and Engineering	1
1.2 Physical Systems, Models, and Representations	2
1.3 Robustness	3
CHAPTER 2 THE SYSTEM REPRESENTATION $R(\cdot) = [A(\cdot), B(\cdot), C(\cdot), D(\cdot)]$	5
2.1 Fundamental Properties of $R(\cdot)$	5
2.1.1 Definitions	5
2.1.2 Structure of $R(\cdot)$	6
2.1.3 State Transition Matrix	10
2.1.4 State Transition Map and Response Map	17
2.1.5 Impulse Response Matrix	22
2.1.6 Adjoint Equations	25
2.1.7 Linear-Quadratic Optimization	29
2.2 Applications	40
2.2.1 Variational Equation	40
2.2.2 Control Correction Example	44
2.2.3 Optimization Example	48
2.2.4 Periodically Varying Differential Equations	51
CHAPTER 2d THE DISCRETE-TIME SYSTEM REPRESENTATION $R_d(\cdot) = [A(\cdot), B(\cdot), C(\cdot), D(\cdot)]$	55
2d.1 Fundamental Properties of $R_d(\cdot)$	58
2d.2 Application: Periodically Varying Recursion Equations	66
CHAPTER 3 THE SYSTEM REPRESENTATION $R = [A, B, C, D]$, Part I	68
3.1 Preliminaries	68
3.2 General Properties of $R = [A, B, C, D]$	70
3.2.1 Definition	70
3.2.2 State Transition Matrix	70

3.2.3 The State Transition and Response Map of R	76
3.3 Properties of R when A has a Basis of Eigenvectors	79
 CHAPTER 3d THE DISCRETE-TIME SYSTEM REPRESENTATION	
$R_d = [A, B, C, D]$	95
3d.1 Preliminaries	95
3d.2 General Properties of R_d	95
3d.3 Properties of R_d when A has a Basis of Eigenvectors	100
 CHAPTER 4 THE SYSTEM REPRESENTATION $R = [A, B, C, D]$, Part II	
4.1 Preliminaries	103
4.2 Minimal Polynomial	107
4.3 Decomposition Theorem	110
4.4 The Decomposition of a Linear Map	117
4.5 Jordan Form	122
4.6 Function of a Matrix	127
4.7 Spectral Mapping Theorem	135
4.8 The Linear Map $X \rightarrow AX + XB$	138
 CHAPTER 5 GENERAL SYSTEM CONCEPTS	
5.1 Dynamical Systems	140
5.2 Time-Invariant Dynamical Systems	150
5.3 Linear Dynamical Systems	151
5.4 Equivalence	152
 CHAPTER 6 SAMPLED DATA SYSTEMS	
6.1 Relation Between L - and z -Transforms	160
6.2 D/A Converter	166
6.3 A/D Converter	167
6.4 Sampled-Data System	168
6.5 Example	171
 CHAPTER 7 STABILITY	
7.1 I/O Stability	173
7.2 State Related Stability Concepts and Applications	180
7.2.1 Stability of $\dot{x} = A(t)x$	180
7.2.2 Bounded Trajectories and Regulation	190
7.2.3 Response to T-Periodic Inputs	193

7.2.4 Periodically Varying System with Periodic Input	196
7.2.5 Slightly Nonlinear Systems	197
CHAPTER 7d STABILITY: THE DISCRETE-TIME CASE	204
7d.1 I/O Stability	204
7d.2 State Related Stability Concepts	211
7d.2.1 Stability of $x(k+1) = A(k)x(k)$	211
7d.2.2 Bounded Trajectories and Regulation	217
7d.2.3 Response to q-Periodic Inputs	220
CHAPTER 8 CONTROLLABILITY AND OBSERVABILITY	222
Introduction	222
8.1 Controllability and Observability of Dynamical Systems	222
8.2 Controllability of the Pair $(A(\cdot), B(\cdot))$	226
8.2.1 Controllability of the Pair $(A(\cdot), B(\cdot))$	226
8.2.2 The Cost of Control	229
8.2.3 Stabilization by Linear State Feedback	231
8.3 Observability of the Pair $(C(\cdot), A(\cdot))$	233
8.4 Duality	235
8.5 Linear Time-Invariant Systems	239
8.5.1 Observability Properties of the Pair (C, A)	240
8.5.2 Controllability of the Pair (A, B)	243
8.6 Kalman Decomposition Theorem	247
8.7 Hidden Modes, Stabilizability, and Detectability	252
8.8 Balanced Representations	260
8.9 Robustness of Controllability	262
CHAPTER 8d CONTROLLABILITY AND OBSERVABILITY: THE DISCRETE- TIME CASE	265
8d.1 Controllability and Observability of Dynamical Systems	265
8d.2 Reachability and Controllability of the Pair $(A(\cdot), B(\cdot))$	265
8d.2.1 Controllability of the Pair $(A(\cdot), B(\cdot))$	265
8d.2.2 The Cost of Control	270
8d.3 Observability of the Pair $(C(\cdot), A(\cdot))$	271
8d.4 Duality	275
8d.5 Linear Time-Invariant Systems	279
8d.5.1 Observability of the Pair (C, A)	281
8d.5.2 Reachability and Controllability of the Pair (A, B)	283

8d.6 Kalman Decomposition Theorem	292
8d.7 Stabilizability and Detectability	292
CHAPTER 9 REALIZATION THEORY	295
9.1 Minimal Realizations	295
9.2 Controllable Canonical Form	306
CHAPTER 10 LINEAR STATE FEEDBACK AND ESTIMATION	315
10.1 Linear State Feedback	315
10.2 Linear Output Injection and State Estimation	323
10.3 State Feedback of the Estimated State	328
10.4 Infinite Horizon Linear Quadratic Optimization	330
10d.4 Infinite Horizon Linear Quadratic Optimization. The Discrete-Time Case	346
CHAPTER 11 UNITY FEEDBACK SYSTEMS	356
11.1 The Feedback System Σ_c	357
11.1.1 State Space Analysis	357
11.1.2 Special Case: R_1 and R_2 have no Unstable Hidden Modes	364
11.1.3 The Discrete-Time Case	367
11.2 Nyquist Criterion	368
11.2.1 The Nyquist Criterion	368
11.2.2 Remarks on the Nyquist Criterion	370
11.2.3 Proof of Nyquist Criterion	372
11.2.4 The Discrete-Time Case	374
11.3 Robustness	374
11.3.1 Robustness With Respect to Plant Perturbations	375
11.3.2 Robustness With Respect to Exogenous Disturbances	376
11.3.3 Robust Regulation	377
11.3.4 Bandwidth-Robustness Tradeoff	379
11.3.5 The Discrete-Time Case	383
11.4 Kharitonov's Theorem	383
11.4.1 Hurwitz Polynomials	384
11.4.2 Kharitonov's Theorem	384
11.5 Robust Stability Under Structured Perturbations	388
11.5.1 General Robustness Theorem	389
11.5.2 Special Case: Affine Maps and Convexity	391
11.5.3 The Discrete Time Case	392

11.6	Stability Under Arbitrary Additive Plant Perturbations	393
11.7	Transmission Zeros	396
11.7.1	Single-Input Single-Output Case	396
11.7.2	Multi-Input Multi-Output Case: Assumptions and Definitions	397
11.7.3	Characterization of the Zeros	399
11.7.4	Application to Unity Feedback Systems	401
APPENDIX A LINEAR MAPS AND MATRIX ANALYSIS		403
A.1	Preliminary Notions	403
A.2	Rings and Fields	405
A.3	Linear Spaces	409
A.4	Linear Maps	415
A.5	Matrix Representation	419
A.5.1	The Concept of Matrix Representation	419
A.5.2	Matrix Representation and Change of Basis	423
A.5.3	Range and Null Space: Rank and Nullity	426
A.5.4	Echelon Forms of a Matrix	429
A.6	Normed Linear Spaces	434
A.6.1	Norms	434
A.6.2	Convergence	437
A.6.3	Equivalent Norms	438
A.6.4	The Lebesgue Spaces l^p and L^p [Tay.1]	440
A.6.5	Continuous Linear Transformations	441
A.7	The Adjoint of a Linear Map	447
A.7.1	Inner Products	448
A.7.2	Adjoint of Continuous Linear Maps	452
A.7.3	Properties of the Adjoint	456
A.7.4	The Finite Rank Operator Fundamental Lemma	457
A.7.5	Singular Value Decomposition (SVD)	459
APPENDIX B DIFFERENTIAL EQUATIONS		469
B.1	Existence and Uniqueness of Solutions	469
B.1.1	Assumptions	469
B.1.2	Fundamental Theorem	470
B.1.3	Construction of a Solution by Iteration	471
B.1.4	The Bellman-Gronwall Inequality	475
B.1.5	Uniqueness	476
B.2	Initial Conditions and Parameter Perturbations	477

B.3 Geometric Interpretation and Numerical Calculations	480
APPENDIX C LAPLACE TRANSFORMS	482
C.1 Definition of the Laplace Transform	482
C.2 Properties of Laplace Transforms	484
APPENDIX D THE z-TRANSFORM	488
D.1 Definition of the z-Transform	488
D.2 Properties of the z-Transform	489
REFERENCES	492
ABBREVIATIONS	498
MATHEMATICAL SYMBOLS	499
SUBJECT INDEX	504