

CONTENTS

Preface	ix
Abbreviations and Symbols	xi
Chapter 1. INTRODUCTION	1
1.1. General Remarks	1
1.2. Orthogonal Functions: Historical Remarks	2
1.3. Applications of Walsh Functions	3
1.4. This Work	3
Selected References	4
Chapter 2. WALSH FUNCTIONS	5
2.1. Introduction	5
2.2. Dyadic Representation of Real Numbers	5
2.2.1. Dyadic expansions	6
2.2.2. Dyadic addition	6
2.2.3. Dyadic shift	8
2.3. Walsh Functions	9
2.4. Properties of Walsh Functions	12
2.5. Remarks on Walsh Series	13
2.5.1. Walsh series expansions	13
2.5.2. Parseval's Relation	17
2.5.3. A translation invariant integral	17
2.5.4. Dyadic correlation of periodic signals	18
2.6. On Computing Walsh Coefficients	19
2.7. A Walsh-Dirichlet Kernel	22
2.8. Harmonic Ordering of Walsh Functions	24
Selected References	27

Chapter 3.	WALSH TRANSFORM	29
3.1.	Introduction	29
3.2.	Generalized Walsh Functions	30
3.3.	The Walsh Transform	32
3.3.1.	Definition	32
3.3.2.	Delta function	32
3.3.3.	Properties of Walsh transform	33
3.3.4.	Some examples	39
3.4.	The Fine Integral	41
3.5.	Relations Between Walsh and Fourier Transforms	43
3.6.	On Computing Generalized Walsh Transforms	44
3.6.1.	Computing from definition	44
3.6.2.	Computing via Fourier transform	45
3.7.	Discrete Walsh and Hadamard Transforms	46
3.8.	Two-Dimensional Discrete Walsh Transform	51
3.9.	Poisson Sum Formulas for Walsh Analysis	51
	Selected References	54
Chapter 4.	CERTAIN CONCEPTS IN STOCHASTIC STUDIES	56
4.1.	Introduction	56
4.2.	Dyadic and Stayadic Correlation	56
4.2.1.	General definitions	56
4.2.2.	Dyadic and stayadic correlations of ergodic processes	58
4.3.	Sequency Spectral Representation	59
4.4.	Walsh Characteristic Functions	62
4.5.	Sampling Theorems	65
4.5.1.	Deterministic case	65
4.5.2.	Further discussion	68
4.5.3.	Stochastic case	71
4.6.	Bounds on Sampling Expansion Truncation Errors	74
4.6.1.	Deterministic case	75
4.6.2.	Stochastic case	77
4.7.	Walsh Sampling Representation for Nonstationary Processes	78
4.7.1.	Sampling representation	78
4.7.2.	Truncation error bounds	82
	Selected References	83
Chapter 5.	LINEAR DYADIC INVARIANT SYSTEMS	85
5.1.	Introduction	85
5.2.	General Analysis	85
5.3.	Dyadic and Stayadic Correlation, and Sequency Spectral Analysis	89
5.4.	Wide Sense Stationary Processes in LDI Systems	90
5.5.	Optimal LDI Systems	91
5.6.	The Dyadic Matched Filter	94
5.7.	Sequency Filters	96
5.7.1.	Generalized ideal sequency filters	96

	5.7.2. Walsh-Harmuth filters	100
	5.7.3. On time-sequence relationship	102
5.8.	Considerations in Discrete LDI Systems	103
	5.8.1. General analysis	104
	5.8.2. A state space approach	107
5.9.	On Identification of LDI Systems	109
5.10.	WDS Processes in Discrete Linear Time Invariant Systems	110
	Selected References	112
Chapter 6.	WALSH ANALYSIS OF NONLINEAR SYSTEMS	114
	6.1. Introduction	114
	6.2. A Walsh Characteristic Function Method	114
	6.3. A Matrix Method	118
	6.4. Quasi-Linear Approximation	123
	6.5. Walsh Analysis of Power-Law Systems	127
	6.5.1. General analysis	127
	6.5.2. A truncated Walsh series expansion	130
	6.6. A Quantizer Analysis	132
	6.6.1. Walsh transform of nonlinearity	133
	6.6.2. Stochastic correlation analysis	133
	6.6.3. On a certain useful integral	134
	6.7. Identification of Nonlinear Systems	136
	6.7.1. General analysis	136
	6.7.2. Determination of the Volterra kernels	139
	6.8. Epilogue	144
	Selected References	146
Chapter 7.	APPLICATIONS OF WALSH FUNCTIONS TO STATISTICAL PROBLEMS	148
	7.1. Introduction	148
	7.2. Statistical Properties of Rademacher Functions	148
	7.3. Statistical Properties of Walsh Functions	151
	7.4. Probability Transformations	152
	7.5. Walsh Series Expansions of Probability Distributions	154
	7.5.1. Walsh expansion of a univariate PDF	156
	7.5.2. Walsh expansion of a bivariate PDF	161
	7.6. Applications to Nonlinear Transformations (Systems)	166
	7.6.1. Some generalities	166
	7.6.2. General moments of nonlinear systems	168
	Selected References	172
Chapter 8.	HAAR FUNCTIONS	174
	8.1. Introduction	174
	8.2. Haar Functions	174
	8.3. Haar Series Expansion	177

8.4.	Relations Between Haar and Walsh Functions	180
8.5.	Discrete Haar Transform	181
8.6.	On an Alternative Definition of Haar Functions	183
8.7.	On Applications of Haar Functions	184
	Selected References	185
Chapter 9.	RELATION BETWEEN WALSH AND FOURIER SPECTRA	187
9.1.	Introduction	187
9.2.	Trigonometric Fourier Series	187
9.3.	Interrelationships of Walsh and Fourier Series	189
9.4.	Relations Between Discrete Walsh and Fourier Transforms	194
9.5.	Comparison of Various Forms of Walsh and Fourier Transforms	196
9.6.	Relations Between the DWT and the z-Transform	200
	Selected References	205
Chapter 10.	SOME APPLICATIONS IN COMMUNICATIONS	207
10.1.	Introduction	207
10.2.	Multiplexing	208
	10.2.1. Time-division multiplexing	208
	10.2.2. Frequency-division multiplexing	210
	10.2.3. Sequence-division multiplexing	211
	10.2.4. Some general remarks on multiplexing	214
10.3.	Binary and Data Coding	215
	10.3.1. Preliminary notions in coding theory	215
	10.3.2. Binary and Walsh codes	216
	10.3.3. Data coding	218
10.4.	Image Transmission	219
	10.4.1. Representation of continuous images	219
	10.4.2. Certain digital image processing aspects	220
10.5.	Signal Processing	226
	10.5.1. ECG analysis	227
	10.5.2. Speech processing	230
10.6.	Epilogue	233
	Selected References	235
Appendix A.	Fast Hadamard Transform: An Algorithm	239
Appendix B.	Kramer's Generalized Sampling Theorem	242
Appendix C.	Discrete Fourier Transform and Properties	244
Appendix D.	Karhunen-Loève Transform (Expansion)	248
	Supplementary Selected References	251
Subject Index		259
Author Index		263