Contents

1. Introduction 1

2. Strategies and payoffs 5

2.1. A general setting for evolutionary game theory 6
2.2. Mixed strategies and population games 8
2.3. Finite number of strategies 13
2.4. Infinitely many (pure) strategies 15
2.5. Structured populations: asymmetric contests and multitype games . . 17
2.6. Additional remarks 21

3. Evolutionary stability 25

3.1. Definition of evolutionary stability 25
3.2. Evolutionary stability and solution concepts in classical game theory . 30
3.3. Conditions for evolutionary stability based on the normal cone 31
3.4. Conditions for evolutionary stability using smoothness 35
3.5. Evolutionary stability in populations with fixed structure 38
3.6. Additional remarks 42

4. Global invasion barriers 45

4.1. Uninvadability: definition and examples 45
4.2. Conditions for uninvadability based on continuity properties 49
4.3. Conditions for uninvadability using smoothness 53
4.4. An illustrative example 56
4.5. Additional remarks 57

5. Games with bilinear average mean payoff 61

5.1. Pairwise conflicts and bilinear average mean payoff 61
5.2. Evolutionary stability under bilinearity 64
5.3. Uninvadability under bilinearity 66

6. Mixed strategies 69

6.1. Evolutionary stability and extremality 70
6.2. The structure of C_P for polyhedral $\overline{X}$ 73
6.3. On the number of evolutionarily stable states 76
6.4. Additional remarks 81

7. Population games 83

7.1. On the equivalence of evolutionary stability and uninvadability 84
7.2. Conditions for evolutionary stability and uninvadability 90
7.3. The case of polyhedral population strategy sets 93
7.4. The role of copositivity in population games with $\overline{X} = S^n$ 95
7.5. Additional remarks 99

8. Replicator dynamics 105

8.1. Dynamical stability and uninvadability 106
8.2. Dynamical versus evolutionary stability 111

Appendix 117

References 139

List of symbols 143