

Contents

Preface	xv
Acknowledgments	xix
Glossary of Symbols	xx
1 Introduction	1
1.1 Dynamic Systems	1
1.1.1 <i>Dynamic Systems with Inputs</i>	3
1.1.2 <i>Time-variant State Equations</i>	4
1.1.3 <i>Continuous State Equations</i>	5
1.1.4 <i>System Definition, State Definition and Modeling</i>	6
1.2 Dynamic Programming	7
1.3 State Variables, Decision Variables and State Equations	8
1.4 Criterion Functions	11
1.5 Optimality	12
1.6 Problem Restatement and Further Discussion	14
1.7 Some Preliminary Remarks on Decision and State Constraints	16
1.8 Other Approaches—Pro and Con	17
1.9 <i>N</i> -Dimensional Spaces	19
1.10 Time-independent (or Static) Problems	20
1.11 Dynamic Programming Revisited	20
1.12 “An Elementary Introduction”	21
2 A Simple Dynamic Optimization Problem and Its Relationship to Other Problems	23
2.1 The Problem	23
2.1.1 <i>Dynamic Programming Solution</i>	25
2.1.2 <i>Discussion of the Solution</i>	31

2.1.3	<i>Separability of the Criterion Function</i>	32
2.1.4	<i>State Separation</i>	33
2.1.5	<i>N-Stage Process: $P_N(x_0)$</i>	34
2.1.6	<i>Simplification of Notation</i>	34
2.1.7	<i>Example of Inductive Proof Using Functional Equations</i>	36
2.1.8	<i>The Equivalence of $P_3(x)$, a Boundary-value Problem, to an Allocation Problem and to a "Static" Mathematical Extremum Problem</i>	38
2.1.9	<i>The Min and Max Operators and Their Manipulation</i>	40
2.2	<i>Max ($v_1^2 + v_2^2 + v_3^2$) Subject to $v_1 + v_2 + v_3 = x_0, v_s \geq 0$, All s</i>	44
2.3	<i>Max ($v_1^2 + v_2^2 + v_3^2$) Subject to $v_1 + v_2 + v_3 \leq x_0, v_s \geq 0$, All s</i>	49
2.4	<i>Discussion</i>	51
2.5	<i>A Quadratic Programming Example</i>	52
2.5.1	<i>Additional Independent Constraints</i>	53
2.5.2	<i>Additional Dependent Constraints</i>	54
2.6	<i>An Integer Programming Example</i>	56
2.7	<i>A Problem with Quadratic Constraints</i>	61
2.7.1	<i>Max ($v_1 + v_2 + v_3$) Subject to $v_1^2 + v_2^2 + v_3^2 = x_0; v_s \geq 0$</i>	61
2.7.2	<i>Min ($v_1 + v_2 + v_3$) Subject to $v_1^2 + v_2^2 + v_3^2 = x_0; v_s \geq 0$</i>	63
2.8	<i>An Example of Multiplicative Separation of the Criterion Function: A Proof that the Arithmetic Mean of N Positive Numbers Is Greater Than or Equal to Their Geometric Mean</i>	64
2.9	<i>The Versatility of Dynamic Programming</i>	66
	<i>Exercises</i>	66

3 Policy and State Spaces, Criterion Functions and Functional Equations

68

3.1	<i>Problem Statement</i>	68
3.2	<i>Decision Spaces</i>	71
3.3	<i>State Spaces</i>	73
3.3.1	<i>Constants, Parameters and State Variables</i>	74
3.4	<i>Criterion Functions</i>	75
3.4.1	<i>Terminal Criterion Functions</i>	76
3.4.2	<i>Path Criterion Functions</i>	77
3.4.3	<i>Mixed Criterion Functions</i>	77
3.5	<i>The Principle of Optimality</i>	78
3.6	<i>Some Functional Equations</i>	80
3.6.1	<i>Terminal Criterion Functions</i>	81
3.6.2	<i>Path Criterion Functions</i>	82
3.6.3	<i>Mixed Criterion Functions</i>	84
3.7	<i>Conditions on the Validity and Usefulness of the Principle of Optimality</i>	84
3.7.1	<i>Validity: Separability of the Criterion Function</i>	85
3.7.2	<i>Usefulness: Markovian State Property</i>	85
3.7.3	<i>State Transformations to Achieve Satisfaction of Condition II</i>	86
3.8	<i>Finite- and Infinite-stage Processes</i>	87
	<i>Exercises</i>	90

4.1	A Simple Problem: P_N	93
4.1.1	Added Independent Decision Constraints: Q_N	96
4.1.2	Added Dependent Decision Constraints: R_N	97
4.1.3	Added State Constraints: $P_{N,K}, Q_{N,K}, R_{N,K}$	98
4.2	The Infinite-stage Problem: P_∞	100
4.2.1	Q_∞	102
4.2.2	R_∞	102
4.2.3	$P_{\infty,K}, Q_{\infty,K}, R_{\infty,K}$	103
4.3	Infinitesimal Time Intervals Between Decisions: $P_\infty(\Delta)$	103
4.4	Path Criterion Functions $\sum_0^N x_s^2, \sum_0^{N_0} h(x_s)$	106
4.5	Terminal Control	106
4.5.1	Infinite-stage Processes	108
4.6	Some Solutions	108
4.6.1	P_N	109
4.6.2	Q_N	110
4.6.3	R_N	113
4.6.4	$P_{N,K}, Q_{N,K}, R_{N,K}$	115
4.6.5	P_∞	120
4.6.6	Q_∞	121
4.6.7	$P_{\infty,K}, Q_{\infty,K}, R_{\infty,K}$	122
4.6.8	$Q_\infty(\Delta)$	122
4.7	Discussion	123
	Exercises	123

5 Computational Solution of Functional Equations

5.1	Tables of Functions $f_i(x)$ and Decisions $v(x)$	125
5.2	Construction of Tables of $f_i(x)$ and $v(x)$: The Discrete Search	128
5.2.1	$f_{r+1}(0)$ and $v(0)$	130
5.2.2	$f_{r+1}(0.01)$ and $v(0.01)$	130
5.2.3	$f_{r+1}(0.02)$ and $v(0.02)$	131
5.2.4	$f_{r+1}(1.98)$ and $v(1.98)$	132
5.2.5	$f_{r+1}(1.99)$ and $v(1.99)$	132
5.2.6	$f_{r+1}(2)$ and $v(2)$	132
5.2.7	Discrete Searches Revisited	133
5.3	Cumulative Errors in Computational Solutions	133
5.4	The Expanding Grid	135
5.5	Expanding State Dimensionality	138
5.6	Computer Storage and Running-Time	140
5.7	Monotonicity, Convexity and Concavity	142
	Exercises	149

6	An Investment-Allocation Problem	151
6.1	Model 1: Investment in Toto	152
6.2	Model 2: Divided Investment	154
6.3	Model 3: Investment in Toto with Broker's Fees	156
6.4	Model 4: Extension of Model 3 to m Alternative Investments	158
6.5	Discussion	161
	<i>Exercises</i>	<i>163</i>
7	Stochastic Multi-stage Decision Processes	164
7.1	Introduction	164
7.2	Comparative Deterministic and Stochastic Processes: An Example	166
	7.2.1 <i>A Deterministic Walk: Relentless Ruin</i>	167
	7.2.2 <i>A Random Walk: Gambler's Ruin (Version 1)</i>	168
	7.2.3 <i>Gambler's Ruin (Version 2)</i>	174
7.3	Some General Remarks	176
	7.3.1 <i>The State Equation</i>	176
	7.3.2 <i>The Criterion Function</i>	178
	7.3.3 <i>Probabilistic Operations on the Criterion Functions</i>	179
7.4	Some Functional Equations	180
	7.4.1 <i>Maximizing and Minimizing Expectations of Criterion Functions</i>	180
	7.4.2 <i>Maximizing and Minimizing Probabilities Associated with Final States</i>	183
7.5	Stochastic Analogies to the Problems of Chapter 2	187
	7.5.1 <i>A Stochastic Criterion Function</i>	187
	7.5.2 <i>A Stochastic State Equation</i>	189
7.6	Stochastic Extensions of Model P_N of Chapter 4	191
	7.6.1 <i>Criterion Function $\sum x_s$</i>	191
	7.6.2 <i>Criterion Function $\sum x_s^2$</i>	192
	7.6.3 <i>Discussion</i>	193
7.7	A Stochastic Version of the Investment-Allocation Problem of Chapter 6 (Investment in Toto)	194
	7.7.1 <i>Maximizing $E(x_N)$</i>	196
	7.7.2 <i>Maximizing prob $(x_N > a^N x_0)$</i>	197
	7.7.3 <i>Discussion</i>	201
	<i>Exercises</i>	<i>201</i>
8	A Stochastic Inventory Model	203
8.1	The Model	205
8.2	The Infinite-stage Process	208
	8.2.1 <i>The Optimal Policy</i>	208

8.2.2	<i>Proof of the Optimal Policy</i>	210
8.2.3	<i>Calculation of $f(x)$</i>	213
8.2.4	<i>Some Further Remarks</i>	214
8.3	The Infinite-stage Process with Continuous Demands	215
8.4	Penalty Cost $p(z - v) + q$, N Infinite	217
8.4.1	<i>Discrete Demands</i>	219
8.4.2	<i>Continuous Demands</i>	220
8.5	The Finite-stage Process Revisited	222
8.6	Some Final Comments	224
8.6.1	<i>Further Variations on the Inventory Model</i>	225
	<i>Exercises</i>	226
9	Adaptive Multi-stage Decision Processes	227
9.1	Deterministic, Stochastic and Adaptive Processes	227
9.2	Bayesian Probabilities	230
9.2.1	<i>Bayesian Probabilities: An Example</i>	230
9.2.2	<i>Bayesian Probabilities: General Remarks</i>	235
9.3	Adaptive Ruin	241
9.3.1	<i>Two Possible Values of p: $p_1 = 0.6$ and $p_2 = 0.4$—Type (i)</i>	242
9.3.2	<i>Two General Possible Values of p: $p = p_1$ and $p = p_2$—Type (i)</i>	245
9.3.3	<i>The Parameter p May Have Any Value Between 0 and 1—Type (iii)</i>	246
9.3.4	<i>The Parameter p Has the Beta Distribution—Type (iii)</i>	248
9.4	An Adaptive Version of the Investment-allocation Problems of Chapters 6 and 7	250
9.4.1	<i>A General Adaptive Investment-allocation Model—Type (iv)</i>	251
9.4.2	<i>Reduction of Adaptive Functional Equations to Manageable Form: Bayesian Invariance</i>	252
9.4.3	<i>Adaptive Investment-allocation with $h(k)$ and $\eta(z)$ Both Normal</i>	254
9.4.4	<i>Adaptive Extensions of Other Investment-allocation Models</i>	258
9.5	An Adaptive Search Problem—Type (i)	259
9.5.1	<i>An Optimal Policy for Detecting a Fault in a Complex System</i>	259
9.5.2	<i>Determination of the Optimal Policy at the First Level</i>	261
9.5.3	<i>Determination of the Optimal Policy at the Second Level</i>	263
	<i>Exercises</i>	264
10	A Sequential Analysis Problem	266
10.1	Mathematical Statistics, Sequential Analysis and Dynamic Programming	266
10.2	Testing of Two Alternative Hypotheses H_1 and H_2	269
10.2.1	<i>Black Boxes</i>	274
10.3	Testing H_1 Versus H_2 Using Dynamic Programming and Considering Trial Costs	275
10.4	Testing H_1 Versus H_2 When the Number of Trials Is Limited	278

10.4.1	<i>Proof of Equation (8)</i>	280
10.4.2	<i>Proof of Equation (9)</i>	282
10.5	Conditions on p_1, p_2, c Under Which No Testing Is Performed	283
10.6	Computational Determination of Optimal Policies	284
10.7	Hypotheses H_1 and H_2 : Normal Distributions with Means μ_1 and μ_2 and Variances 1	285
10.8	Determination of a and b as Analytical Functions	287
10.9	Discussion	287
	<i>Exercises</i>	288

11 Optimal Trajectories and Control Theory

289

11.1	Control Theory	289
11.2	Optimal Trajectories	289
11.3	The Quadratic Terminal-regulator Problem	291
11.3.1	<i>A Stochastic Version</i>	293
11.4	The Quadratic State-regulator Problem	294
11.4.1	<i>A Stochastic Version</i>	295
11.5	Further Examples of Motion of a Particle Along a Straight Line	295
11.5.1	<i>Some Continuous Versions of Motion Along a Straight Line</i>	299
11.6	Motion in a Plane; Random Walk in a Square	301
11.6.1	<i>Random Walks and Diffusion Processes</i>	304
11.6.2	<i>Minimum Time Problem: Minimizing $E(T)$</i>	306
11.6.3	<i>Minimizing $E \sum_0^T (x_s - y_s)^2$</i>	313
11.7	Motion in a Plane: Travel from a Flat Earth to a Flat Moon	315
11.8	Invariant Imbedding	320
11.9	Discussion	320
	<i>Exercises</i>	321

12 Simulation and Dynamic Programming

322

12.1	Simulation of Multi-stage Decision Processes	322
12.2	Simulation, Optimal Policies and Sub-Optimal Policies	326
12.3	Estimation of $f(x)$ When the Optimal Policy Has Been Derived Without Calculating $f(x)$	332
12.3.1	(i) <i>The Process Q_N of Chapter 4</i>	333
12.3.2	(ii) <i>The Infinite-stage Inventory Process of Chapter 8</i>	334
12.3.3	(iii) <i>The Adaptive Ruin Process of Chapter 9</i>	336
12.3.4	(iv) <i>The Sequential Analysis Model of Chapter 10</i>	337
12.4	Simulation and Sub-optimality	340
12.4.1	<i>Some Final Remarks</i>	345
	<i>Exercises</i>	346

13 Approximation Techniques

348

13.1	Introduction	348
------	--------------	-----

13.2	Successive Approximation in Policy Space for Processes of Unconstrained Length	349	
13.2.1	<i>Successive Approximation in Policy Space</i>	355	
13.2.2	<i>Some Further Remarks on Convergence</i>	356	
13.3	The Use of Infinite-stage Processes in Approximating the Solution of Finite-stage Processes	358	
13.4	Functional Approximation	358	
13.5	Quasilinearization	363	
13.6	Conclusion	364	
	<i>Exercises</i>	365	
14	Reduction of State Dimensionality Using Lagrange Multipliers		366
14.1	Introduction	366	
14.2	Lagrange Multipliers	367	
14.3	Discussion	370	
14.4	Example I	372	
14.5	Example II	374	
14.6	Example III	375	
14.7	Example IV: A Deterministic Inventory Example	376	
14.8	Inequality Constraints	378	
	<i>Exercise</i>	379	
	Bibliography		380
	Answers to Exercises		382
	Index		395