

Preface	xix
Part I Basic Thoughts and Equations	
<hr/>	
<b>1 Philosophy of Computational Fluid Dynamics</b>	<b>3</b>
1.1 Computational Fluid Dynamics: Why?	4
1.2 Computational Fluid Dynamics as a Research Tool	6
1.3 Computational Fluid Dynamics as a Design Tool	9
1.4 The Impact of Computational Fluid Dynamics—Some Other Examples	13
1.4.1 Automobile and Engine Applications	14
1.4.2 Industrial Manufacturing Applications	17
1.4.3 Civil Engineering Applications	19
1.4.4 Environmental Engineering Applications	20
1.4.5 Naval Architecture Applications (Submarine Example)	22
1.5 Computational Fluid Dynamics: What Is It?	23
1.6 The Purpose of This Book	32
<b>2 The Governing Equations of Fluid Dynamics: Their Derivation, a Discussion of Their Physical Meaning, and a Presentation of Forms Particularly Suitable to CFD</b>	<b>37</b>
2.1 Introduction	38
2.2 Models of the Flow	40
2.2.1 Finite Control Volume	41
2.2.2 Infinitesimal Fluid Element	42
2.2.3 Some Comments	42
2.3 The Substantial Derivative (Time Rate of Change Following a Moving Fluid Element)	43
2.4 The Divergence of the Velocity: Its Physical Meaning	47
2.4.1 A Comment	48
	<b>xi</b>

2.5	The Continuity Equation	49
2.5.1	Model of the Finite Control Volume Fixed in Space	49
2.5.2	Model of the Finite Control Volume Moving with the Fluid	51
2.5.3	Model of an Infinitesimally Small Element Fixed in Space	53
2.5.4	Model of an Infinitesimally Small Fluid Element Moving with the Flow	55
2.5.5	All the Equations Are One: Some Manipulations	56
2.5.6	Integral versus Differential Form of the Equations: An Important Comment	60
2.6	The Momentum Equation	60
2.7	The Energy Equation	66
2.8	Summary of the Governing Equations for Fluid Dynamics: With Comments	75
2.8.1	Equations for Viscous Flow (the Navier–Stokes Equations)	75
2.8.2	Equations for Inviscid Flow (the Euler Equations)	77
2.8.3	Comments on the Governing Equations	78
2.9	Physical Boundary Conditions	80
2.10	Forms of the Governing Equations Particularly Suited for CFD: Comments on the Conservation Form, Shock Fitting, and Shock Capturing	82
2.11	Summary Problems	92 93
<b>3</b>	<b>Mathematical Behavior of Partial Differential Equations: The Impact on CFD</b>	95
3.1	Introduction	95
3.2	Classification of Quasi-Linear Partial Differential Equations	97
3.3	A General Method of Determining the Classification of Partial Differential Equations: The Eigenvalue Method	102
3.4	General Behavior of the Different Classes of Partial Differential Equations: Impact on Physical and Computational Fluid Dynamics	105
3.4.1	Hyperbolic Equations	106
3.4.2	Parabolic Equations	111
3.4.3	Elliptic Equations	117
3.4.4	Some Comments: The Supersonic Blunt Body Problem Revisited	119
3.5	Well-Posed Problems	120
3.6	Summary Problems	121 121

## Part II Basics of the Numerics

---

<b>4</b>	<b>Basic Aspects of Discretization</b>	125
4.1	Introduction	125
4.2	Introduction to Finite Differences	128

4.3	Difference Equations	142
4.4	Explicit and Implicit Approaches: Definitions and Contrasts	145
4.5	Errors and an Analysis of Stability	153
4.5.1	Stability Analysis: A Broader Perspective	165
4.6	Summary	165
	<i>GUIDEPOST</i>	166
	Problems	167
<b>5</b>	<b>Grids with Appropriate Transformations</b>	168
5.1	Introduction	168
5.2	General Transformation of the Equations	171
5.2	Metrics and Jacobians	178
5.4	Form of the Governing Equations Particularly Suited for CFD Revisited: The Transformed Version	183
5.5	A Comment	186
5.6	Stretched (Compressed) Grids	186
5.7	Boundary-Fitted Coordinate Systems; Elliptic Grid Generation	192
	<i>GUIDEPOST</i>	193
5.8	Adaptive Grids	200
5.9	Some Modern Developments in Grid Generation	208
5.10	Some Modern Developments in Finite-Volume Mesh Generation: Unstructured Meshes and a Return to Cartesian Meshes	210
5.11	Summary	212
	Problems	215
<b>6</b>	<b>Some Simple CFD Techniques: A Beginning</b>	216
6.1	Introduction	216
6.2	The Lax-Wendroff Technique	217
6.3	MacCormack's Technique	222
	<i>GUIDEPOST</i>	223
6.4	Some Comments: Viscous Flows, Conservation Form, and Space Marching	225
6.4.1	Viscous Flows	225
6.4.2	Conservation Form	225
6.4.3	Space Marching	226
6.5	The Relaxation Technique and Its Use with Low-Speed Inviscid Flow	229
6.6	Aspects of Numerical Dissipation and Dispersion; Artificial Viscosity	232
6.7	The Alternating-Direction-Implicit (ADI) Technique	243
6.8	The Pressure Correction Technique: Application to Incompressible Viscous Flow	247
6.8.1	Some Comments on the Incompressible Navier-Stokes Equations	248

6.8.2	Some Comments on Central Differencing of the Incompressible Navier–Stokes Equations; The Need for a Staggered Grid	250
6.8.3	The Philosophy of the Pressure Correction Method	253
6.8.4	The Pressure Correction Formula	254
6.8.5	The Numerical Procedure: The SIMPLE Algorithm	261
6.8.6	Boundary Conditions for the Pressure Correction Method	262
	<i>GUIDEPOST</i>	264
6.9	Some Computer Graphic Techniques Used in CFD	264
6.9.1	<i>xy</i> Plots	264
6.9.2	Contour Plots	265
6.9.3	Vector and Streamline Plots	270
6.9.4	Scatter Plots	273
6.9.5	Mesh Plots	273
6.9.6	Composite Plots	274
6.9.7	Summary on Computer Graphics	274
6.10	Summary Problems	277
		278

## Part III Some Applications

---

<b>7</b>	<b>Numerical Solutions of Quasi-One-Dimensional Nozzle Flows</b>	283
7.1	Introduction: The Format for Chapters in Part III	283
7.2	Introduction to the Physical Problem: Subsonic-Supersonic Isentropic Flow	285
7.3	CFD Solution of Subsonic-Supersonic Isentropic Nozzle Flow: MacCormack's Technique	288
	7.3.1 The Setup	288
	7.3.2 Intermediate Results: The First Few Steps	308
	7.3.3 Final Numerical Results: The Steady-State Solution	313
7.4	CFD Solution of Purely Subsonic Isentropic Nozzle Flow	325
	7.4.1 The Setup: Boundary and Initial Conditions	327
	7.4.2 Final Numerical Results: MacCormack's Technique	330
	7.4.3 The Anatomy of a Failed Solution	325
7.5	The Subsonic-Supersonic Isentropic Nozzle Solution Revisited: The Use of the Governing Equations in Conservation Form	336
	7.5.1 The Basic Equations in Conservation Form	337
	7.5.2 The Setup	340
	7.5.3 Intermediate Calculations: The First Time Step	345
	7.5.4 Final Numerical Results: The Steady State Solution	351

7.6	A Case with Shock Capturing	356
7.6.1	The Setup	358
7.6.2	The Intermediate Time-Marching Procedure: The Need for Artificial Viscosity	363
7.6.3	Numerical Results	364
7.7	Summary	372
<b>8</b>	<b>Numerical Solution of a Two-Dimensional Supersonic Flow: Prandtl-Meyer Expansion Wave</b>	374
8.1	Introduction	374
8.2	Introduction to the Physical Problem: Prandtl-Meyer Expansion Wave—Exact Analytical Solution	376
8.3	The Numerical Solution of a Prandtl-Meyer Expansion Wave Flow Field	377
8.3.1	The Governing Equations	377
8.3.2	The Setup	386
8.3.3	Intermediate Results	397
8.3.4	Final Results	407
8.4	Summary	414
<b>9</b>	<b>Incompressible Couette Flow: Numerical Solutions by Means of an Implicit Method and the Pressure Correction Method</b>	416
9.1	Introduction	416
9.2	The Physical Problem and Its Exact Analytical Solution	417
9.3	The Numerical Approach: Implicit Crank-Nicholson Technique	420
9.3.1	The Numerical Formulation	421
9.3.2	The Setup	425
9.3.3	Intermediate Results	426
9.3.4	Final Results	430
9.4	Another Numerical Approach: The Pressure Correction Method	435
9.4.1	The Setup	436
9.4.2	Results	442
9.5	Summary	445
	Problem	446
<b>10</b>	<b>Supersonic Flow over a Flat Plate: Numerical Solution by Solving the Complete Navier–Stokes Equations</b>	447
10.1	Introduction	447
10.2	The Physical Problem	449
10.3	The Numerical Approach: Explicit Finite-Difference Solution of the Two-Dimensional Complete Navier–Stokes Equations	450
10.3.1	The Governing Flow Equations	450
10.3.2	The Setup	452

10.3.3	The Finite-Difference Equations	453
10.3.4	Calculation of Step Sizes in Space and Time	455
10.3.5	Initial and Boundary Conditions	457
10.4	Organization of Your Navier–Stokes Code	459
10.4.1	Overview	459
10.4.2	The Main Program	461
10.4.3	The MacCormack Subroutine	463
10.4.4	Final Remarks	466
10.5	Final Numerical Results: The Steady State-Solution	466
10.6	Summary	474

## Part IV Other Topics

---

<b>11</b>	<b>Some Advanced Topics in Modern CFD:</b>	
	<b>A Discussion</b>	479
11.1	Introduction	479
11.2	The Conservation Form of the Governing Flow Equations Revisited: The Jacobians of the System	480
11.2.1	Specialization to One-Dimensional Flow	482
11.2.2	Interim Summary	489
11.3	Additional Considerations for Implicit Methods	489
11.3.1	Linearization of the Equations: The Beam and Warming Method	490
11.3.2	The Multidimensional Problem: Approximate Factorization	492
11.3.3	Block Tridiagonal Matrices	496
11.3.4	Interim Summary	497
11.4	Upwind Schemes	497
11.4.1	Flux-Vector Splitting	500
11.4.2	The Godunov Approach	502
11.4.3	General Comment	507
11.5	Second-Order Upwind Schemes	507
11.6	High-Resolution Schemes: TVD and Flux Limiters	509
11.7	Some Results	510
11.8	Multigrid Method	513
11.9	Summary	514
	Problems	514
<b>12</b>	<b>The Future of CFD</b>	515
12.1	The Importance of CFD Revisited	515
12.2	Computer Graphics in CFD	516
12.3	The Future of CFD: Enhancing the Design Process	517
12.4	The Future of CFD: Enhancing Understanding	526
12.5	Conclusion	533

<b>Appendix A Thomas' Algorithm for the Solution of a Tridiagonal System of Equations</b>	534
<b>References</b>	539
<b>Index</b>	543