

Contents

1 The Taylor Experiment

1.1	Modeling of the experiment	1
1.1.1	Introduction	1
1.1.2	Mathematical description of the experiment	6
1.1.3	Narrow gap limit and Rayleigh-Bénard problem	8
1.1.4	End effects	11
1.2	Flows between rotating cylinders	13
1.3	Stability of Couette flow	17
1.3.1	Equations of motion for axisymmetric perturbations	19
1.3.2	Computation of marginal curves	23
1.3.3	Validity of the principle of exchange of stability	27

2 Details of a Numerical Method

2.1	Introduction	29
2.1.1	Numerical model	29
2.1.2	Numerical methods	32
2.1.3	Validity of the model	33
2.1.4	Stability	35
2.2	The discretized system	36
2.2.1	Discretization in the axial z -direction	36
2.2.2	Discretization in the radial r -direction	40
2.2.3	Boundary conditions	40
2.2.4	Final version of the equations	42
2.3	Computation of solutions	45
2.3.1	Pseudo-arclength continuation and Newton iterations	45
2.3.2	Continuation in the Reynolds number Re	48

2.3.3	Continuation in the wave number k	50
2.3.4	Simple continuation	51
2.3.5	Switching branches	52
2.4	Computation of flow parameters	55
2.4.1	Periodicity	56
2.4.2	Computation of $u_m := u(r_m, z_m; \lambda)$ at $r_m := 1 + \delta/2$, $z_m = 0$	58
2.4.3	Computation of the torque	59
2.4.4	Computation of kinetic energies	60
2.4.5	Computation of the stream function	64
2.5	Numerical accuracy	66
2.5.1	Finite Differences	67
2.5.2	Truncation of the Fourier Series	71
2.5.3	Conclusions	76
3	Stationary Taylor Vortex Flows	
3.1	Introduction	77
3.2	Computations with fixed period $\lambda \approx 2$	81
3.2.1	A narrow gap problem, $\eta = 0.95$	82
3.2.2	A wide gap problem, $\eta = 0.5$	84
3.3	Variation of flows with period λ	85
3.3.1	Previous results on flows with wavelengths $\lambda \neq 2$	85
3.3.2	Continuous change of period	88
3.3.3	Flows near $\text{Re} = \sqrt{1.5} \text{Re}_{cr}$	91
3.3.4	Flows for $\text{Re} = 800 \approx 3.65 \text{Re}_{cr}$	101
3.4	Interactions of secondary branches	106
3.4.1	A neighborhood of $(\text{Re}_{24}, \lambda_{24})$ and the basic $(2, 4)$ fold	106
3.4.2	Connections to the Rayleigh-Bénard problem	110
3.4.3	The basic $(n, 2n)$ -fold for higher Reynolds numbers	113
3.4.4	The basic 2-vortex surface	117
3.5	$\text{Re} = 2 \text{Re}_{cr}$ and the (n, pn) double points	121
3.6	Stability of the stationary vortices	133
3.6.1	Wavy vortices	133
3.6.2	Eckhaus and short-wavelength instabilities	136

4 Secondary Bifurcations on Convection Rolls

4.1 Introduction 143

4.2 The Rayleigh-Bénard problem 148

 4.2.1 Convection in fluids 148

 4.2.2 Boussinesq approximation 150

 4.2.3 The Rayleigh-Bénard problem as limiting case
 of the Taylor problem 154

4.3 Stationary convection rolls 160

 4.3.1 The basic equations 160

 4.3.2 Critical curves of the primary solution 164

 4.3.3 Pure-mode solutions 167

4.4 The (2,4) interaction in a model problem 169

 4.4.1 The model problem 169

 4.4.2 Calculation of secondary bifurcation points
 on the 2-roll solutions 170

 4.4.3 Calculation of secondary bifurcation points
 on the 4-roll solutions 172

 4.4.4 The perturbation approach 175

4.5 The (2,6) interaction in a model problem 179

 4.5.1 Calculation of secondary bifurcation points
 on the 2-roll solutions 179

 4.5.2 Calculation of secondary bifurcation points
 on the 6-roll solutions 181

 4.5.3 Nonlinear interactions between
 the bifurcating branches 182

4.6 Generalisations and consequences 184

 4.6.1 Other interactions 184

 4.6.2 Linear superpositions of pure-mode solutions 185

 4.6.3 Secondary bifurcations in the
 Taylor problem revisited 188

Bibliography 191

Index 205