

Contents

Chapter I. The gamma function and related functions	1
1.1 The gamma function	1
1.2 The function $\psi(z)$	13
1.3 The Riemann zeta function $\zeta(z)$	19
1.4 The generalized zeta function $\zeta(z, \alpha)$	22
1.5 Bernoulli and Euler polynomials	25
1.6 Lerch's transcendent $\Phi(z, s, \alpha)$	32
1.7 Miscellaneous results	35
Literature	36
Chapter II. The hypergeometric function	37
2.1 Definitions and elementary relations	37
2.2 The hypergeometric differential equation	42
2.3 Gauss' contiguous relations	46
2.4 Linear and higher order transformations	47
2.5 Integral representations	54
2.6 Asymptotic expansions	56
2.7 The Riemann differential equation	57
2.8 Transformation formulas for Riemann's P -function	58
2.9 The generalized hypergeometric series	62
2.10 Miscellaneous results	64
Literature	65
Chapter III. Bessel functions	65
3.1 Solutions of the Bessel and the modified Bessel differential equation	65
3.2 Bessel functions of integer order	69
3.3 Half odd integer order	72
3.4 The Airy functions and related functions	75
3.5 Differential equations and a power series expansion for the product of two Bessel functions	77
3.6 Integral representations for Bessel, Neumann and Hankel functions	79
3.7 Integral representations for the modified Bessel functions	84
3.8 Integrals involving Bessel functions	86
3.9 Addition theorems	106
3.10 Functions related to Bessel functions	108
3.11 Polynomials related to Bessel functions	120
3.12 Series of arbitrary functions in terms of Bessel functions	123
3.13 A list of series involving Bessel functions	129

3.14 Asymptotic expansions	188
3.15 Zeros	146
3.16 Miscellaneous	148
Literature	151
 Chapter IV. Legendre functions	151
4.1 Legendre's differential equation	151
4.2 Relations between Legendre functions	164
4.3 The functions $P_v^u(x)$ and $Q_v^u(x)$. (Legendre functions on the cut)	166
4.4 Special values for the parameters	172
4.5 Series involving Legendre functions	178
4.6 Integral representations	184
4.7 Integrals involving Legendre functions	191
4.8 Asymptotic behavior	195
4.9 Associated Legendre functions and surface spherical harmonics	198
4.10 Gegenbauer functions, toroidal functions and conical functions	199
Literature	203
 Chapter V. Orthogonal polynomials	204
5.1 Orthogonal systems	204
5.2 Jacobi polynomials	209
5.3 Gegenbauer or ultraspherical polynomials	218
5.4 Legendre Polynomials	227
5.5 Generalized Laguerre polynomials	239
5.6 Hermite polynomials	249
5.7 Chebychev (Tchebichef) polynomials	256
Literature	262
 Chapter VI. Kummer's function	262
6.1 Definitions and some elementary results	262
6.2 Recurrence relations	267
6.3 The differential equation	268
6.4 Addition and multiplication theorems	271
6.5 Integral representations	274
6.6 Integral transforms associated with ${}_1F_1(a; c; z)$, $U(a, c, z)$	278
6.7 Special cases and its relation to other functions	283
6.8 Asymptotic expansions	288
6.9 Products of Kummer's functions	293
Literature	295
 Chapter VII. Whittaker function	295
7.1 Whittaker's differential equation	295
7.2 Some elementary results	301
7.3 Addition and multiplication theorems	306
7.4 Integral representations	311
7.5 Integral transforms	314
7.6 Asymptotic expansions	317
7.7 Products of Whittaker functions	321
Literature	323

Chapter VIII. Parabolic cylinder functions and parabolic functions	323
8.1 Parabolic cylinder functions	323
8.2 Parabolic functions	333
Literature	335
Appendix to Chapter VIII	336
Chapter IX. The incomplete gamma function and special cases	337
9.1 The incomplete gamma function	337
9.2 Special cases	342
Literature	357
Chapter X. Elliptic integrals, theta functions and elliptic functions	357
10.1 Elliptic integrals	358
10.2 The theta functions	371
10.3 Definition of the Jacobian elliptic functions by the theta functions	377
10.4 The Jacobian zeta function	386
10.5 The elliptic functions of Weierstrass	387
10.6 Connections between the parameters and special cases	392
Literature	395
Chapter XI. Integral transforms	395
Examples for the Fourier cosine transform	396
Examples for the Fourier sine transform	397
Examples for the exponential Fourier transform	397
Examples for the Laplace transform	397
Examples for the Mellin transform	397
Examples for the Hankel transform	397
Examples for the Lebedev, Mehler and generalised Mehler transform	398
Example for the Gauss transform	398
11.1 Several examples of solution of integral equations of the first kind	465
Literature	467
Appendix to Chapter XI	467
Chapter XII. Transformation of systems of coordinates	472
12.1 General transformation and special cases	472
12.2 Examples of separation of variables	485
Literature	492
List of special symbols	493
List of functions	495
Index	500