

Contents

1.	The Continuum Description	1
1.1	Densities and Fluxes	1
1.2	Conservation and Balance Laws in One Dimension	3
1.3	Heat Flow	7
1.4	Steady Radial Flow in Two Dimensions	10
1.5	Steady Radial Flow in Three Dimensions	13
2.	Unsteady Heat Flow	17
2.1	Thermal Energy	17
2.1.1	Heat Balance in One-dimensional Problems	18
2.1.2	Some Special Solutions of Equation (2.3)	20
2.2	Effects of Heat Supply	26
2.3	Unsteady, Spherically Symmetric Heat Flow	30
3.	Fields and Potentials	35
3.1	Gradient of a Scalar	35
3.1.1	Some Applications	36
3.2	Gravitational Potential	38
3.2.1	Special Properties of the Function $\phi = r^{-1}$	40
3.3	Continuous Distributions of Mass	44
3.4	Electrostatics	46
3.4.1	Gauss's Law of Flux	48
3.4.2	Charge-free Regions	48
3.4.3	Surface Charge Density	50

4. Laplace's Equation and Poisson's Equation	55
4.1 The Ubiquitous Laplacian	55
4.2 Separable Solutions	58
4.3 Poisson's Equation	66
4.4 Dipole Solutions	70
4.4.1 Uses of Dipole Solutions to $\nabla^2\phi = 0$	72
4.4.2 Spherical Inclusions	73
5. Motion of an Elastic String	77
5.1 Tension and Extension; Kinematics and Dynamics	77
5.1.1 Dynamics	79
5.2 Planar Motions	80
5.2.1 Small Transverse Motions	82
5.2.2 Longitudinal Motions	82
5.3 Properties of the Wave Equation	82
5.3.1 Standing Waves	84
5.3.2 Superposition of Standing Waves	86
5.4 D'Alembert's Solution, Travelling Waves and Wave Reflections	90
5.4.1 Wave Reflections	90
5.5 Other One-dimensional Waves	93
5.5.1 Acoustic Vibrations in a Tube	93
5.5.2 Telegraphy and High-voltage Transmission	96
6. Fluid Flow	101
6.1 Kinematics and Streamlines	101
6.1.1 Some Important Examples of Steady Flow	102
6.2 Volume Flux and Mass Flux	103
6.2.1 Incompressible Fluids	105
6.2.2 Mass Conservation	106
6.3 Two-dimensional Flows of Incompressible Fluids	107
6.3.1 The Continuity Equation	107
6.3.2 Irrotational Flows and the Velocity Potential	108
6.3.3 The Stream Function	111
6.4 Pressure in a Fluid	116
6.4.1 Resultant Force	116
6.4.2 Hydrostatics and Archimedes' Principle	117
6.4.3 Momentum Density and Momentum Flux	119
6.5 Bernoulli's Equation	122
6.5.1 The Material (Advection) Derivative	122
6.5.2 Bernoulli's Equation and Dynamic Pressure	123
6.5.3 The Principle of Aerodynamic Lift	125
6.6 Three-dimensional, Incompressible Flows	127

6.6.1	The Continuity Equation	128
6.6.2	Irrotational Flows, the Velocity Potential and Laplace's Equation	129
7.	Elastic Deformations	133
7.1	The Kinematics of Deformation	133
7.1.1	Deformation Gradient	135
7.1.2	Stretch and Rotation	136
7.2	Polar Decomposition	140
7.3	Stress	143
7.3.1	Traction Vectors	144
7.3.2	Components of Stress	144
7.3.3	Traction on a General Surface	146
7.4	Isotropic Linear Elasticity	147
7.4.1	The Constitutive Law	148
7.4.2	Stretching, Shear and Torsion	149
8.	Vibrations and Waves	157
8.1	Wave Reflection and Refraction	157
8.1.1	Use of the Complex Exponential	157
8.1.2	Plane Waves	159
8.1.3	Reflection at a Rigid Wall	161
8.1.4	Refraction at an Interface	163
8.1.5	Total Internal Reflection	165
8.2	Guided Waves	167
8.2.1	Acoustic Waves in a Layer	167
8.2.2	Waveguides and Dispersion	169
8.3	Love Waves in Elasticity	174
8.4	Elastic Plane Waves	176
8.4.1	Elastic Shear Waves	176
8.4.2	Dilatational Waves	178
9.	Electromagnetic Waves and Light	185
9.1	Physical Background	185
9.1.1	The Origin of Maxwell's Equations	185
9.1.2	Plane Electromagnetic Waves	188
9.1.3	Reflection and Refraction of Electromagnetic Waves	192
9.2	Waveguides	195
9.2.1	Rectangular Waveguides	196
9.2.2	Circular Cylindrical Waveguides	198
9.2.3	An Introduction to Fibre Optics	202

10. Chemical and Biological Models	207
10.1 Diffusion of Chemical Species	207
10.1.1 Fick's Law of Diffusion	208
10.1.2 Self-similar Solutions	209
10.1.3 Travelling Wavefronts	211
10.2 Population Biology	214
10.2.1 Growth and Dispersal	214
10.2.2 Fisher's Equation and Self-limitation	216
10.2.3 Population-dependent Dispersivity	219
10.2.4 Competing Species	221
10.2.5 Diffusive Instability	224
10.3 Biological Waves	229
10.3.1 The Logistic Wavefront	229
10.3.2 Travelling Pulses and Spiral Waves	231
Solutions	237
Bibliography	261
Index	263