

# Table of Contents

Introduction . . . . .	1
Why Model? . . . . .	2

## 1. Foundations of Modeling Dynamic Systems

1.1. Time . . . . .	4
1.2. Dynamics . . . . .	5
1.3. State . . . . .	5
1.4. Discrete and Continuous Representations of Time . . . . .	6
1.5. The Discrete Nature of Observed Time and Observed States . . . . .	7
1.6. State Spaces . . . . .	8
1.7. Progress Through State Space . . . . .	11
1.8. The Conditional Probability of Transition from State to State . . . . .	14
1.9. Network Representations of Primitive Markovian State Spaces . . . . .	18
1.10. Conservation . . . . .	21
1.11. State Variables Associated with Individual Organisms . . . . .	22
1.12. Basic Analysis of Markov Chains . . . . .	24
1.13. Vector Notation, State Projection Matrices . . . . .	27
1.14. Elementary Dynamics of Homogeneous Markov Chains . . . . .	30
1.15. Observation of Transition Probabilities . . . . .	43
1.16. The Primitive State Space for an Entire Population of Identical Objects . . . . .	46
1.17. Dynamics of Populations Comprising Indistinguishable Members . . . . .	49
1.18. Deduction of Population Dynamics Directly from the Member State Space . . . . .	55
1.19. A Situation in Which Member State Space Cannot be Used to Deduce Population Dynamics . . . . .	58
1.20. The Law of Large Numbers . . . . .	61
1.21. Summary . . . . .	62
1.22. Some References for Chapter 1 . . . . .	64

## 2. General Concepts of Population Modeling

2.1. Lumped Markovian States from Irreducible Primitive Markovian State Spaces . . . . .	65
2.2. Shannon's Measure: Uncertainty in State Spaces and Lumped States . . . . .	71
2.3. Lumped Markovian States from Reducible Primitive Markovian State Spaces . . . . .	76

2.4.	Frequency Aliasing: The Artifact of Lumped Time . . . . .	87
2.5.	Idealizations: Thought Experiments and Hypothesis Testing . . . . .	90
2.6.	Conservation: Defining Membership in a Given Population . . . . .	91
2.7.	Conservation and Constitutive Relationships for a Single State . . . . .	93
2.8.	Reproduction, Death and Life as Flow Processes . . . . .	96
2.9.	Further Lumping: Combining Age Classes for Simplified Situations and Hypotheses . . . . .	100
2.10.	The Use of Network Diagrams to Construct Models . . . . .	102
2.11.	Basic Principles of Network Construction . . . . .	111
2.12.	Some Alternative Representations of Common Network Configura- tions . . . . .	122
2.13.	Some References for Chapter 2 . . . . .	126

### 3. A Network Approach to Population Modeling

3.1.	Introduction to Network Modeling of Populations . . . . .	128
3.2.	Network Models for Some Basic, Idealized Life Cycles . . . . .	129
3.2.1.	Simple Life Cycles . . . . .	130
3.2.2.	Models with Overlapping Time Classes . . . . .	144
3.3.	<i>Scalor</i> Parameters and <i>Multiplier</i> Functions . . . . .	150
3.3.1.	Single-Species Models . . . . .	151
3.3.2.	Two-Species Models . . . . .	163
3.4.	<i>Time-Delay</i> Durations . . . . .	176
3.5.	Conversion to a Stochastic Model . . . . .	183
3.5.1.	Models without Interactions of Correlated Variables . . . . .	185
3.5.2.	Models with Interactions of Correlated Variables . . . . .	187
3.5.3.	Examples of Stochastic Network Models . . . . .	190
3.6.	Some References for Chapter 3 . . . . .	197

### 4. Analysis of Network Models

4.1.	Introduction to Network Analysis . . . . .	198
4.2.	Interval by Interval Accounting on a Digital Computer . . . . .	200
4.2.1.	Digital Representation of <i>Time Delays</i> in Large-Numbers Mo- dels . . . . .	200
4.2.2.	Digital Representations for Stochastic Network Models . . . . .	205
4.2.3.	Examples of Digital Modeling . . . . .	208
4.3.	Graphical Analysis of One-Loop Networks with Lumpable Param- eters . . . . .	209
4.4.	Large-Numbers Models with Constant Parameters . . . . .	225
4.5.	Inputs and Outputs of Network Models . . . . .	226
4.6.	Linearity, Cohorts, and Superposition-Convolution . . . . .	227
4.7.	The $z$ -Transform: A Shorthand Notation for Discrete Functions . . . . .	235
4.8.	The Application of $z$ -Transforms to Linear Network Functions . . . . .	238
4.8.1.	Straight-Chain Networks . . . . .	240
4.8.2.	Networks with Feed Forward Loops . . . . .	241
4.8.3.	Networks with Feedback Loops . . . . .	241

4.9.	Linear Flow-Graph Analysis . . . . .	247
4.9.1.	Graphical Reduction of Flow Graphs . . . . .	248
4.9.2.	Mason's Rule . . . . .	258
4.10.	Interpretation of Unit-Cohort Response Functions: The Inverse $z$ -Transform . . . . .	263
4.10.1.	Partial-Fraction Expansion . . . . .	265
4.10.2.	Finding the Coefficients of an Expansion . . . . .	268
4.10.3.	Dealing with Multiple Roots at the Origin . . . . .	272
4.10.4.	Exercises . . . . .	275
4.11.	Types of Common Ratios and Their Significances . . . . .	275
4.11.1.	The General Nature of Common Ratios . . . . .	276
4.11.2.	Real Common Ratios . . . . .	278
4.11.3.	Complex Common Ratios . . . . .	280
4.11.4.	Exercises . . . . .	286
4.12.	The Patterns of Linear Dynamics . . . . .	286
4.12.1.	Growth Patterns in Low-Level Populations and Populations with Constant Parameters . . . . .	287
4.12.2.	Time Required for Establishment of Dominance . . . . .	288
4.12.3.	Geometric Patterns of Growth and the Biotic Potential . . . . .	293
4.12.4.	Patterns of Dynamics Near Nonzero Critical Levels . . . . .	295
4.12.5.	Exercises . . . . .	296
4.13.	Constant-Parameter Models for Nonzero Critical Levels . . . . .	296
4.13.1.	Replacement for the <i>Scalor</i> . . . . .	296
4.13.2.	Modification of the <i>Time Delay</i> . . . . .	300
4.13.3.	Dynamics Close to a Nonzero Critical Level . . . . .	300
4.14.	Finding the Roots of $Q(z)$ . . . . .	302
4.14.1.	Euclid's Algorithm . . . . .	305
4.14.2.	Descartes' Rule and Sturm Sequences . . . . .	309
4.14.3.	Locating the Real Roots . . . . .	313
4.14.4.	Locating Imaginary and Complex Roots . . . . .	317
4.14.5.	Estimating the Magnitude of the Dominant Common Ratio . . . . .	321
4.15.	Network Responses to More Complicated Input Patterns . . . . .	328
4.15.1.	The Natural Frequencies of a Constant-Parameter Network Model . . . . .	329
4.15.2.	Exciting and Observing the Natural Frequencies of a Network . . . . .	330
4.15.3.	Responses to Category-1 Inputs . . . . .	333
4.15.4.	The Initial- and Final-Value Theorems, Steady-State Analysis . . . . .	338
4.16.	Elements of Dynamic Control of Networks . . . . .	341
4.16.1.	Linear Control with Category-2 Inputs . . . . .	342
4.16.2.	Comments on Nonlinear Control and Regulation . . . . .	347
4.17.	Dynamics of Constant-Parameter Models with Stochastic <i>Time Delays</i> . . . . .	348
4.17.1.	$z$ -Transforms of Stochastic <i>Time Delays</i> . . . . .	349
4.17.2.	Moments of Stochastic <i>Time Delay</i> Distributions . . . . .	350
4.17.3.	Examples of Stochastic <i>Time Delays</i> and Their Transforms . . . . .	351
4.17.4.	Effects of <i>Time Delay</i> Distributions on Dynamics in a One-Loop Model . . . . .	351

4.17.5. An Elementary Sensitivity Analysis . . . . .	354
4.17.6. When the Minimum Latency is Not Finite . . . . .	357
4.18. The Inverse Problem: Model Synthesis . . . . .	358
4.19. Application of Constant-Parameter Network Analysis to More Gen- eral Homogeneous Markov Chains . . . . .	360
4.20. Some References for Chapter 4 . . . . .	365
Appendix A. Probability Arrays, Array Manipulation . . . . .	367
A.1. Definitions . . . . .	367
A.2. Manipulation of Arrays . . . . .	370
A.3. Operations on Probability Arrays . . . . .	375
Appendix B. Bernoulli Trials and the Binomial Distribution . . . . .	378
Bibliography . . . . .	384
Subject Index . . . . .	394