## Contents

1	Introduction		
	1.1	A first look at multilevel systems analysis	1
	1.2	The multilevel character of systems	3
	1.3	The historical development of multilevel systems analysis	4
	1.4	Overview of the volume	6
		References	9
2	Fundamental Concepts		
	2.1	The idealized multilevel approach	10
		2.1.1 Partitioning the overall problem into subproblems, 10	
		2.1.2 Examples of two-level subproblem hierarchies, 12	
		2.1.3 The multilevel solution process, 17	
	2.2	Additional aspects of multilevel systems analysis	19
		2.2.1 A more general multilevel approach, 19	
		2.2.2 Institutional interpretations and analogies, 22	
		2.2.3 Related concepts, 23	
		References	25
3	Multilevel Solution Methods 2		
	3.1	Introduction and overview	
	3.2	Column generation	28
		3.2.1 General discussion, 28	
		3.2.2 The maximal multicommodity network flow problem, 31	
		3.2.3 Solution by column generation, 33	
	3.3 The Dantzig-Wolfe decomposition method for linear		
		programs	36
		3.3.1 The representation of a polyhedral convex set, 37	

			An outline of the Dantzig–Wolfe decomposition method, 37	
			Linear programming problems with unbounded	
			solutions, 41	
		3.3.4	A numerical example, 43	
		3.3.5	Some further remarks on the Dantzig-Wolfe	
			decomposition method, 45	
		3.3.6	Block-angular structures, 48	
	3.4	The Dar	ntzig-Wolfe method for nonlinear programs	54
	3.5	The Benders algorithm and some extensions		56
			An outline of the Benders algorithm, 56	
		3.5.2	A note on Step 2 of the Benders algorithm, 61	
		3.5.3	A numerical example, 63	
		3.5.4	The application of the Benders algorithm to block-	
			angular structures, 66	
		3.5.5	On the relation between the Benders and Dantzig-	
			Wolfe algorithms, 69	
	3.6	The Kor	rnai-Liptak decomposition algorithm	70
	3.7	Lagrang	ean decomposition in nonlinear programming	72
			Lagrangean decomposition for separable	
			nathematical programming problems, 73	
			Duality theory and Lagrangean decomposition, 75	
	3.8		c methods	76
	3.9		el control theory: a brief survey	76
			Static multilevel control problems, 77	
			Dynamic open-loop multilevel control, 78	
			On-line control models, 81	
		Referen	ces	81
4	The state of the s			
4.1 On the utilization of structure in solving				
			ming problems	84
	4.2		blem experiences	86
		Reference	ces	97
5				99
	5.1 Introduction and overview			99
	5.2	Multilev	el national economic planning in Hungary	101
			The application of the Kornai–Liptak method to a	
			ational economic planning problem, 101	
		5.2.2 $T$	he application of man-machine planning to the	
			966–1970 5-year plan, 105	
		5.2.3 C	Concluding remarks, 111	

	5.3	Multilevel national economic planning in Mexico 5.3.1 Introduction, 112	112	
		5.3.2 DINAMICO, 115		
		•		
		· · · · · · · · · · · · · · · · · · ·		
		5.3.4 Linkages between DINAMICO and ENERGETICOS, 120 5.3.5 Conclusions and comparison with multilevel national economic planning in Hungary, 123		
	5.4	A problem of regional planning	125	
		5.4.1 The development network, 125		
		5.4.2 An LP model for resource production, 127		
		5.4.3 The overall problem and a two-level solution method, 1	28	
		5.4.4 Discussion of the two-level method for regional planning, 129		
		References	130	
6	Plan	lanning of Production and Sales Programs in Corporations		
	6.1	Introduction	132	
		6.1.1 The planning problem, 132		
		6.1.2 Planning procedures based on decomposition methods,	133	
	6.2	A simulation study of a planning procedure based on the		
		Dantzig-Wolfe method in a paperboard factory	137	
		6.2.1 The planning problem of the paperboard factory, 137		
		6.2.2 Information dispersal and information flows, 140		
		6.2.3 The simulation experiment, 141		
		6.2.4 Implementation of the plan, 142		
		6.2.5 Some conclusions, 144		
	6.3	A simulation study of planning procedures based on the		
		Dantzig-Wolfe and ten Kate methods in a slaughterhouse	145	
		6.3.1 The planning problem of the slaughterhouse, 145		
		6.3.2 Simulated results using the Dantzig-Wolfe method as		
		a planning procedure, 147		
		6.3.3 Simulated results using the ten Kate method as a		
		planning procedure, 149		
		6.3.4 Some conclusions, 152		
	6.4	Final remarks on planning procedures based on		
		decomposition methods	153	
		References	154	
7	0	anti	156	
1		rations Management	156	
	7.1	Introduction and overview	158	
	7.2	A column generation approach	130	
		7.2.1 An approximating LP problem, 158		

		7.2.2 Generation of dominant schedules and a two-level algorithm, 159	
		7.2.3 Applications, 161	
	7.3	Hierarchical production planning	162
	,,,	7.3.1 Introduction to hierarchical production planning, 16.	
		7.3.2 A three-level disaggregation scheme, 162	٤
		7.3.3 The product-type-level subproblem, 163	
		7.3.4 The item-family-level subproblems, 164	
		7.3.5 The item-level subproblems, 168	
		7.3.6 A three-level solution procedure, 168	
		7.3.7 Applications and a comparison with column	
		generation, 170	
		References	171
8	Diet	tribution Systems	150
O	8.1	Introduction and overview	172
	8.2	The optimal design of a distribution system	172
	0.2	8.2.1 A mixed-integer programming formulation, 173	173
		8.2.2 Application of the Benders algorithm, 174	
		8.2.3 The implementation of Geoffrion and Graves, 176	
	8.3	Determining optimal production—distribution programs	179
		8.3.1 A network flow formulation, 179	1/9
		8.3.2 A column generation method, 180	
		8.3.3 The implementation of Folie and Tiffin, 181	
		References	181
9.	Ema	ight Chin Doute Cabadalina and Elasticia Const	100
9.		right Ship Route Scheduling and Electricity Generation	183
		Introduction and overview	183
	9.2	Freight ship route scheduling	183
		9.2.1 Problem formulation, 183	
		9.2.2 The generation of ship itineries, 185	
		9.2.3 A column generation scheme, 187	
	9.3	9.2.4 A three-level method, 189	100
	9.3	Planning power generation	190
		9.3.1 Problem formulation, 190 9.3.2 Application of the Dantzig-Wolfe method, 191	
		9.3.2 Application of the Dantzig-Wolfe method, 191 References	102
		References	193
10		er Pollution Control	194
		Introduction and overview	194
	10.2	The Miami River case	195
		10.2.1 The overall problem, 195	

			xi
		10.2.2 A planning procedure based on Dantzig-Wolfe decomposition, 197	
		10.2.3 A Lagrangean solution method, 200	
	10.3	Concluding remarks	202
		References	203
11	Conclusion		205
	11.1	Problem structures and solution methods	205
	11.2	An evaluation of the usefulness of multilevel methods	208
	11.3	A final word	213
		References	213
	Index		215