

Table of Contents

Chapter 1 Introduction	1
1.1 Conservation Laws and Auxiliary Relations.....	2
1.1.1 Conservation Laws	2
1.1.2 Auxiliary Relations.....	3
1.2 Properties and Categories of Balances.....	3
1.2.1 Dependent and Independent Variables	5
1.2.2 Integral and Differential Balances: The Role of Balance Space and Geometry	5
1.2.3 Unsteady-State Balances: The Role of Time	5
1.2.4 Steady-State Balances.....	7
1.2.5 Dependence on Time and Space	7
1.3 Three Physical Configurations	7
1.3.1 The Stirred Tank	7
1.3.2 The One-Dimensional Pipe	8
1.3.3 The Quenched Steel Billet	9
1.4 Types of ODE and AE Mass Balances	9
1.5 Information Obtained from Model Solutions	10
1.5.1 Steady-State Integral Balances	10
1.5.2 Steady-State One-Dimensional Differential Balances	11
1.5.3 Unsteady Instantaneous Integral Balances.....	11
1.5.4 Unsteady Cumulative Integral Balances	11
1.5.5 Unsteady Differential Balances.....	12
1.5.6 Steady Multidimensional Differential Balances	12
Illustration 1.1 Design of a Gas Scrubber.....	13
Illustration 1.2 Flow Rate to a Heat Exchanger.....	14
Illustration 1.3 Fluidization of a Particle.....	14
Illustration 1.4 Evaporation of Water from an Open Trough	15
Illustration 1.5 Sealing of Two Plastic Sheets.....	15
Illustration 1.6 Pressure Drop in a Rectangular Duct.....	16
Practice Problems	16
References.....	17
Chapter 2 The Setting Up of Balances	19
Illustration 2.1 The Surge Tank	19
Illustration 2.2 The Steam-Heated Tube.....	22
Illustration 2.3 Design of a Gas Scrubber Revisited.....	24
Illustration 2.4 An Example from Industry: Decontamination of a Nuclear Reactor Coolant.....	26

Illustration 2.5 Thermal Treatment of Steel Strapping	29
Illustration 2.6 Batch Filtration: The Ruth Equations.....	32
Illustration 2.7 Drying of a Nonporous Plastic Sheet.....	35
Practice Problems	38
References.....	42
Chapter 3 More About Mass, Energy, and Momentum Balances.....	45
3.1 The Terms in the Various Balances.....	45
3.2 Mass Balances	46
3.2.1 Molar Mass Flow in Binary Mixtures	46
3.2.2 Transport Coefficients.....	48
Illustration 3.2.1 Drying of a Plastic Sheet Revisited:	
Estimation of the Mass Transfer Coefficient k_y	52
Illustration 3.2.2 Measurement of Diffusivities by the	
Two-Bulb Method: The Quasi-Steady State.....	55
3.2.3 Chemical Reaction Mass Balance.....	57
Illustration 3.2.3 CSTR with Second Order Homogeneous	
Reaction $A + B \rightarrow P$	57
Illustration 3.2.4 Isothermal Tubular Reactor with First	
Order Homogeneous Reaction.....	59
Illustration 3.2.5 Isothermal Diffusion and First Order	
Reaction in a Spherical, Porous Catalyst Pellet:	
The Effectiveness Factor E	60
3.2.4 Tank Mass Balance.....	62
Illustration 3.2.6 Waste-Disposal Holding Tank	63
Illustration 3.2.7 Holding Tank with Variable Holdup.....	64
3.2.5 Tubular Mass Balances.....	65
Illustration 3.2.8 Distillation in a Packed Column: The Case	
of Total Reflux and Constant α	66
Illustration 3.2.9 Tubular Flow with Solute Release from	
the Wall	68
Practice Problems	69
3.3 Energy Balances	71
3.3.1 Energy Flux	71
3.3.2 Transport Coefficients.....	72
Illustration 3.3.1 Heat Transfer Coefficient in a Packed Bed	
of Metallic Particles	75
Illustration 3.3.2 The Counter-Current Single Pass Shell	
and Tube Heat Exchanger.....	76
Illustration 3.3.3 Response of a Thermocouple to a	
Temperature Change	82
Illustration 3.3.4 The Longitudinal, Rectangular Heat	
Exchanger Fin	83
Illustration 3.3.5 A Moving Bed Solid-Gas Heat	
Exchanger.....	86

	Illustration 3.3.6 Conduction Through a Hollow Cylinder: Optimum Insulation Thickness.....	89
	Illustration 3.3.7 Heat-Up Time of an Unstirred Tank.....	92
	Illustration 3.3.8 The Boiling Pot.....	94
	Illustration 3.3.9 Melting of a Silver Sample: Radiation.....	96
	Illustration 3.3.10 Adiabatic Compression of an Ideal Gas: Energy Balance for Closed Systems: First Law of Thermodynamics.....	99
	Illustration 3.3.11 The Steady-State Energy Balance for Flowing (Open) Systems.....	101
	Illustration 3.3.12 A Moving Boundary Problem: Freeze-Drying of Food.....	102
	Practice Problems.....	105
3.4	Force and Momentum Balances.....	110
	3.4.1 Momentum Flux and Equivalent Forces.....	110
	3.4.2 Transport Coefficients.....	110
	Illustration 3.4.1 Forces on Submerged Surfaces: Archimedes' Law.....	114
	Illustration 3.4.2 Forces Acting on a Pressurized Container: The Hoop-Stress Formula.....	116
	Illustration 3.4.3 The Effects of Surface Tension: Laplace's Equation; Capillary Rise.....	117
	Illustration 3.4.4 The Hypsometric Formulae.....	120
	Illustration 3.4.5 Momentum Changes in a Flowing Fluid: Forces on a Stationary Vane.....	121
	Illustration 3.4.6 Particle Movement in a Fluid.....	123
	Illustration 3.4.7 The Bernoulli Equation: Some Simple Applications.....	128
	Illustration 3.4.8 The Mechanical Energy Balance.....	132
	Illustration 3.4.9 Viscous Flow in a Parallel Plate Channel: Velocity Distribution and Flow Rate — Pressure Drop Relation.....	135
	Illustration 3.4.10 Non-Newtonian Fluids.....	136
	Practice Problems.....	139
3.5	Combined Mass and Energy Balances.....	142
	Illustration 3.5.1 Nonisothermal CSTR with Second Order Homogeneous Reaction $A + B \rightarrow P$	142
	Illustration 3.5.2 Nonisothermal Tubular Reactors: The Adiabatic Case.....	143
	Illustration 3.5.3 Heat Effects in a Catalyst Pellet: Maximum Pellet Temperature.....	145
	Illustration 3.5.4 The Wet-Bulb Temperature.....	149
	Illustration 3.5.5 Humidity Charts: The Psychrometric Ratio...	151
	Illustration 3.5.6 Operation of a Water Cooling Tower.....	157
	Illustration 3.5.7 Design of a Gas Scrubber Revisited: The Adiabatic Case.....	160

	Illustration 3.5.8 Flash Vaporization.....	162
	Illustration 3.5.9 Steam Distillation.....	165
	Practice Problems	167
3.6	Combined Mass, Energy, and Momentum Balances	172
	Illustration 3.6.1 Isothermal Compressible Flow in a Pipe	173
	Illustration 3.6.2 Propagation of a Pressure Wave, Velocity of Sound, Mach Number	174
	Illustration 3.6.3 Adiabatic Compressible Flow in a Pipe.....	177
	Illustration 3.6.4 Compressible Flow Charts.....	179
	Illustration 3.6.5 Compressible Flow in Variable Area Ducts with Friction and Heat Transfer.....	181
	Illustration 3.6.6 The Converging-Diverging Nozzle	183
	Illustration 3.6.7 Forced Convection Boiling: Vaporizers and Evaporators.....	184
	Illustration 3.6.8 Film Condensation on a Vertical Plate	188
	Illustration 3.6.9 The Nonisothermal, Nonisobaric Tubular Gas Flow Reactor.....	191
	Practice Problems	196
	References.....	198
Chapter 4	Ordinary Differential Equations.....	203
4.1	Definitions and Classifications	203
	4.1.1 Order of an ODE	203
	4.1.2 Linear and Nonlinear ODEs.....	205
	4.1.3 ODEs with Variable Coefficients	206
	4.1.4 Homogeneous and Nonhomogeneous ODEs	207
	4.1.5 Autonomous ODEs.....	208
	Illustration 4.1.1 Classification of Model ODEs.....	208
4.2	Boundary and Initial Conditions	209
	4.2.1 Some Useful Hints on Boundary Conditions	211
	Illustration 4.2.1 Boundary Conditions in a Conduction Problem: Heat Losses from a Metallic Furnace Insert	212
4.3	Analytical Solutions of ODEs.....	213
	4.3.1 Separation of Variables.....	216
	Illustration 4.3.1 Solution of Complex ODEs by Separation of Variables	217
	Illustration 4.3.2 Repeated Separation of Variables: The Burning Fuel Droplet as a Moving Boundary Problem.....	218
	4.3.2 The D-Operator Method: Solution of Linear nth Order ODEs with Constant Coefficients.....	221
	Illustration 4.3.3 The Longitudinal Heat Exchanger Fin Revisited.....	223
	Illustration 4.3.4 Polymer Sheet Extrusion: The Uniformity Index.....	225
	4.3.3 Nonhomogeneous Linear Second Order ODEs with Constant Coefficients	230

	Illustration 4.3.5 Vibrating Spring with a Forcing Function	230
4.3.4	Series Solutions of Linear ODEs with Variable Coefficients	232
	Illustration 4.3.6 Solution of a Linear ODE with Constant Coefficients by a Power Series Expansion	233
	Illustration 4.3.7 Evaluation of a Bessel Function	235
	Illustration 4.3.8 Solution of a Second Order ODE with Variable Coefficients by the Generalized Formula	238
	Illustration 4.3.9 Concentration Profile and Effectiveness Factor of a Cylindrical Catalyst Pellet	239
4.3.5	Other Methods	240
	Illustration 4.3.10 Product Distribution in Reactions in Series: Use of the Substitution $y = vx$	241
	Illustration 4.3.11 Path of Pursuit	243
	Illustration 4.3.12 Design of a Parabolic Mirror	244
4.4	Numerical Methods	245
4.4.1	Boundary Value Problems	246
4.4.2	Initial Value Problems	246
4.4.3	Sets of Simultaneous Initial Value ODEs	249
4.4.4	Potential Difficulties: Stability	249
	Illustration 4.4.1 Example of a Solution by Euler's Method	250
	Illustration 4.4.2 Solution of Two Simultaneous ODEs by the Runge-Kutta Method	251
4.5	Nonlinear Analysis	252
4.5.1	Phase Plane Analysis: Critical Points	253
	Illustration 4.5.1 Analysis of the Pendulum	255
4.5.2	Analysis in Parameter Space: Bifurcations, Multiplicities, and Catastrophe	258
	Illustration 4.5.2 Bifurcation Points in a System of Nonlinear Algebraic Equations	262
	Illustration 4.5.3 A System with a Hopf Bifurcation	263
4.5.3	Chaos	265
	Practice Problems	268
	References	270
Chapter 5 The Laplace Transformation		273
5.1	General Properties of the Laplace Transform	274
	Illustration 5.1.1 Inversion of Various Transforms	278
5.2	Application to Differential Equations	280
	Illustration 5.2.1 The Mass Spring System Revisited: Resonance	282
	Illustration 5.2.2 Equivalence of Mechanical Systems and Electrical Circuits	284
	Illustration 5.2.3 Response of First Order Systems	286
	Illustration 5.2.4 Response of Second Order Systems	290
	Illustration 5.2.5 The Horizontal Beam Revisited	296

5.3	Block Diagrams: A Simple Control System	298
5.3.1	Water Heater	301
5.3.2	Measuring Element.....	301
5.3.3	Controller and Control Element.....	302
5.4	Overall Transfer Function; Stability Criterion; Laplace Domain Analysis	302
	Illustration 5.4.1 Laplace Domain Stability Analysis	305
	Practice Problems	307
	References.....	310
Chapter 6	Special Topics	313
6.1	Biomedical Engineering, Biology, and Biotechnology.....	314
	Illustration 6.1.1 One-Compartment Pharmacokinetics	314
	Illustration 6.1.2 Blood–Tissue Interaction as a Pseudo One-Compartment Model	319
	Illustration 6.1.3 A Distributed Model: Transport Between Flowing Blood and Muscle Tissue	321
	Illustration 6.1.4 Another Distributed Model: The Krogh Cylinder.....	322
	Illustration 6.1.5 Membrane Processes: Blood Dialysis	324
	Illustration 6.1.6 Release or Consumption of Substances at the Blood Vessel Wall.....	330
	Illustration 6.1.7 A Simple Cellular Process.....	333
	Illustration 6.1.8 Turing’s Paper on Morphogenesis	338
	Illustration 6.1.9 Biotechnology: Enzyme Kinetics	341
	Illustration 6.1.10 Cell Growth, Monod Kinetics, Steady-State Analysis of Bioreactors.....	344
	Practice Problems	348
6.2	A Visit to the Environment.....	351
	Illustration 6.2.1 Mercury Volatilization from Water.....	353
	Illustration 6.2.2 Rates of Volatilization of Solutes from Aqueous Solutions	356
	Illustration 6.2.3 Bioconcentration in Fish.....	357
	Illustration 6.2.4 Cleansing of a Lake Bottom Sediment	359
	Illustration 6.2.5 The Streeter-Phelps River Pollution Model: The Oxygen Sag Curve.....	361
	Illustration 6.2.6 Contamination of a River Bed (Equilibrium).....	364
	Illustration 6.2.7 Clearance of a Contaminated River Bed (Equilibrium).....	366
	Illustration 6.2.8 Minimum Bed Requirements for Adsorptive Water Purification (Equilibrium)	367
	Illustration 6.2.9 Actual Bed Requirements for Adsorptive Water Purification (Nonequilibrium).....	368
	Practice Problems	371

6.3	Welcome to the Real World	373
	Illustration 6.3.1 Production of Heavy Water by Methane Distillation.....	373
	Illustration 6.3.2 Clumping of Coal Transported in Freight Cars.....	377
	Illustration 6.3.3 Pop Goes the Vessel.....	378
	Illustration 6.3.4 Debugging of a Vinyl Chloride Recovery Unit.....	379
	Illustration 6.3.5 Pop Goes the Vessel (Again).....	383
	Illustration 6.3.6 Potential Freezing of a Water Pipeline.....	385
	Illustration 6.3.7 Failure of Heat Pipes	387
	Illustration 6.3.8 Coating of a Pipe	389
	Illustration 6.3.9 Release of Potentially Harmful Chemicals to the Atmosphere	392
	Illustration 6.3.10 Design of a Marker Particle (Revisited).....	396
	Practice Problems	398
	References.....	404

Chapter 7 Partial Differential Equations: Classification, Types, and Properties; Some Simple Transformations and Solutions 407

7.1	Properties and Classes of PDEs	409
7.1.1	Order of a PDE.....	409
	7.1.1.1 First Order PDEs	409
	7.1.1.2 Second Order PDEs.....	410
	7.1.1.3 Higher Order PDEs	410
7.1.2	Homogeneous PDEs and BCs.....	410
7.1.3	PDEs with Variable Coefficients	411
7.1.4	Linear and Nonlinear PDEs: A New Category — Quasilinear PDEs	411
7.1.5	Another New Category: Elliptic, Parabolic, and Hyperbolic PDEs	412
7.1.6	Boundary and Initial Conditions	413
	Illustration 7.1.1 Classification of PDEs	415
	Illustration 7.1.2 Derivation of Boundary and Initial Condition.....	416
7.2	PDEs of Major Importance	418
7.2.1	First Order Partial Differential Equations.....	419
	7.2.1.1 Unsteady Tubular Operations (Turbulent Flow).....	419
	7.2.1.2 The Chromatographic Equations.....	419
	7.2.1.3 Stochastic Processes	421
	7.2.1.4 Movement of Traffic.....	421
	7.2.1.5 Sedimentation of Particles.....	422
7.2.2	Second Order Partial Differential Equations	422
	7.2.2.1 Laplace's Equation	422
	7.2.2.2 Poisson's Equation.....	426

7.2.2.3	Helmholtz Equation	427
7.2.2.4	Biharmonic Equation	427
7.2.2.5	Fourier's Equation	428
7.2.2.6	Fick's Equation	428
7.2.2.7	The Wave Equation	428
7.2.2.8	The Navier-Stokes Equations	429
7.2.2.9	The Prandtl Boundary Layer Equations	430
7.2.2.10	The Graetz Problem	431
	Illustration 7.2.1 Derivation of Some Simple PDEs	431
7.3	Useful Simplifications and Transformations	435
7.3.1	Elimination of Independent Variables: Reduction to ODEs	435
7.3.1.1	Separation of Variables	436
7.3.1.2	Laplace Transform	437
7.3.1.3	Similarity or Boltzmann Transformation: Combination of Variables	437
	Illustration 7.3.1 Heat Transfer in Boundary Layer Flow over a Flat Plate: Similarity Transformation	438
7.3.2	Elimination of Dependent Variables: Reduction of Number of Equations	443
	Illustration 7.3.2 Use of the Stream Function in Boundary Layer Theory: Velocity Profiles Along a Flat Plate	443
7.3.3	Elimination of Nonhomogeneous Terms	445
	Illustration 7.3.3 Conversion of a PDE to Homogeneous Form	445
7.3.4	Change in Independent Variables: Reduction to Canonical Form ..	447
	Illustration 7.3.4 Reduction of ODEs to Canonical Form	450
7.3.5	Simplification of Geometry	454
7.3.5.1	Reduction of a Radial Spherical Configuration into a Planar One	456
7.3.5.2	Reduction of a Radial Circular or Cylindrical Configuration into a Planar One	456
7.3.5.3	Reduction of a Radial Circular or Cylindrical Configuration to a Semi-Infinite One	457
7.3.5.4	Reduction of a Planar Configuration to a Semi-Infinite One	457
7.3.6	Nondimensionalization	457
	Illustration 7.3.5 Nondimensionalization of Fourier's Equation	457
7.4	PDEs PDQ: Locating Solutions in Related Disciplines; Solution by Simple Superposition Methods	459
7.4.1	In Search of a Literature Solution	460
	Illustration 7.4.1 Pressure Transients in a Semi-Infinite Porous Medium	460
	Illustration 7.4.2 Use of Electrostatic Potentials in the Solution of Conduction Problems	463

7.4.2	Simple Solutions by Superposition	464
7.4.2.1	Superposition by Simple Flows: Solutions in Search of a Problem	464
	Illustration 7.4.3 Superposition of Uniform Flow and a Doublet: Flow Around an Infinite Cylinder or a Circle.....	468
7.4.2.2	Superposition by Multiplication: Product Solutions.....	470
7.4.2.3	Solution of Source Problems: Superposition by Integration	472
	Illustration 7.4.4 The Instantaneous Infinite Plane Source	474
	Illustration 7.4.5 Concentration Distributions from a Finite and Instantaneous Plane Pollutant Source in Three-Dimensional Semi-Infinite Space.....	479
7.4.2.4	More Superposition by Integration: Duhamel's Integral and the Superposition of Danckwerts	482
	Illustration 7.4.6 A Problem with the Design of Xerox Machines	483
	Practice Problems	488
	References.....	492
Chapter 8	Vector Calculus: Generalized Transport Equations.....	495
8.1	Vector Notation and Vector Calculus	496
8.1.1	Synopsis of Vector Algebra.....	496
	Illustration 8.1.1 Two Geometry Problems	501
8.1.2	Differential Operators and Vector Calculus	503
8.1.2.1	The Gradient ∇	505
8.1.2.2	The Divergence $\nabla \cdot$	506
8.1.2.3	The Curl $\nabla \times$	507
8.1.2.4	The Laplacian ∇^2	508
	Illustration 8.1.2 Derivation of the Divergence.....	510
	Illustration 8.1.3 Derivation of Some Relations Involving ∇ , $\nabla \cdot$, and $\nabla \times$	511
8.1.3	Integral Theorems of Vector Calculus.....	512
	Illustration 8.1.4 Derivation of the Continuity Equation	513
	Illustration 8.1.5 Derivation of Fick's Equation.....	514
	Illustration 8.1.6 Superposition Revisited: Green's Functions and the Solution of PDEs by Green's Functions	515
	Illustration 8.1.7 The Use of Green's Functions in Solving Fourier's Equation.....	520
	Practice Problems	523
8.2	Transport of Mass.....	526
	Illustration 8.2.1 Catalytic Conversion in a Coated Tubular Reactor: Locating Equivalent Solutions in the Literature.....	527
	Illustration 8.2.2 Diffusion and Reaction in a Semi-Infinite Medium: Another Literature Solution	532
	Illustration 8.2.3 The Graetz-Lévéque Problem in Mass Transfer: Transport Coefficients in the Entry Region.....	533

	Illustration 8.2.4 Unsteady Diffusion in a Sphere: Sorption and Desorption Curves.....	538
	Illustration 8.2.5 The Sphere in a Well-Stirred Solution: Leaching of a Slurry	540
	Illustration 8.2.6 Steady-State Diffusion in Several Dimensions.....	542
	Practice Problems	543
8.3	Transport of Energy.....	545
	Illustration 8.3.1 The Graetz-L�ev�eque Problem (Yet Again!) ...	546
	Illustration 8.3.2 A Moving Boundary Problem: Freezing in a Semi-Infinite Solid.....	549
	Illustration 8.3.3 Heat Transfer in a Packed Bed: Heat Regenerators.....	551
	Illustration 8.3.4 Unsteady Conduction.....	554
	Illustration 8.3.5 Steady-State Temperatures and Heat Flux in Multidimensional Geometries: The Shape Factor.....	556
	Practice Problems	556
8.4	Transport of Momentum.....	560
	Illustration 8.4.1 Steady, Fully Developed Incompressible Duct Flow.....	562
	Illustration 8.4.2 Creeping Flow.....	564
	Illustration 8.4.3 The Prandtl Boundary Layer Equations.....	565
	Illustration 8.4.4 Inviscid Flow: Euler's Equations of Motion	567
	Illustration 8.4.5 Irrotational (Potential) Flow: Bernoulli's Equation	568
	Practice Problems	569
	References.....	571
Chapter 9 Solution Methods for Partial Differential Equations		575
9.1	Separation of Variables.....	575
9.1.1	Orthogonal Functions and Fourier Series	575
9.1.1.1	Orthogonal and Orthonormal Functions	580
	Illustration 9.1.1 The Cosine Set.....	582
9.1.1.2	The Sturm-Liouville Theorem.....	583
9.1.1.3	Fourier Series.....	583
	Illustration 9.1.2 Fourier Series Expansion of a Function f(x).....	585
	Illustration 9.1.3 The Quenched Steel Billet Revisited.....	586
	Illustration 9.1.4 Conduction in a Cylinder with External Resistance: Arbitrary Initial Distribution	591
	Illustration 9.1.5 Steady-State Conduction in a Hollow Cylinder.....	593
	Practice Problems	597
9.2	Laplace Transformation and Other Integral Transforms	599
9.2.1	General Properties	599

9.2.2	The Role of the Kernel.....	601
9.2.3	Pros and Cons of Integral Transforms	604
	9.2.3.1 Advantages.....	604
	9.2.3.2 Disadvantages	605
9.2.4	The Laplace Transformation of PDEs	605
	Illustration 9.2.1 Inversion of a Ratio of Hyperbolic Functions	606
	Illustration 9.2.2 Conduction in a Semi-Infinite Medium.....	607
	Illustration 9.2.3 Conduction in a Slab: Solution for Small Time Constants	609
	Illustration 9.2.4 Conduction in a Cylinder Revisited: Use of Hankel Transforms	611
	Illustration 9.2.5 Analysis in the Laplace Domain: The Method of Moments	614
	Practice Problems	617
9.3	The Method of Characteristics	620
	9.3.1 General Properties	620
	9.3.2 The Characteristics	622
	Illustration 9.3.1 The Heat Exchanger with a Time-Varying Fluid Velocity	625
	Illustration 9.3.2 Saturation of a Chromatographic Column	627
	Illustration 9.3.3 Elution of a Chromatographic Column	630
	Illustration 9.3.4 Development of a Chromatographic Pulse.....	632
	Illustration 9.3.5 A Traffic Problem	634
	Practice Problems	636
	References.....	637
	Index.....	639