

# Contents

<b>Chapter 1. Fecundability</b> . . . . .	1
1.1 Introduction . . . . .	1
1.2 A Model of Constant Fecundability – The Geometric Distribution . . . . .	1
1.3 Applying the Geometric Distribution to Data . . . . .	3
1.4 A Model of Heterogeneous Fecundability . . . . .	6
1.5 Some Properties of the Beta-Geometric Distribution . . . . .	8
1.6 Applying the Beta-Geometric Distribution to Data . . . . .	10
1.7 An Investigation of Selectivity . . . . .	13
1.8 Fecundability as a Function of Coital Pattern . . . . .	14
1.9 A Distribution on the Set of Coital Patterns . . . . .	17
1.10 Some Implications of Markov Chain Model of Coital Patterns . . . . .	20
1.11 Computer Implementation and Numerical Examples . . . . .	22
1.12 Conclusions and Further Research Directions . . . . .	26
Problems and Miscellaneous Complements . . . . .	28
References . . . . .	33
<b>Chapter 2. Human Survivorship</b> . . . . .	35
2.1 Introduction . . . . .	35
2.2 Mortality in a Cohort . . . . .	36
2.3 Simple Parametric Examples of the Force of Mortality . . . . .	37
Examples: 2.3.1 The Exponential Distribution 37. – 2.3.2 The Weibull Distribution 38. – 2.3.3 The Gompertz Distribution 38	
2.4 Period Mortality – A Simple Algorithm . . . . .	39
2.5 Transforming Central Death Rates into Probabilities and Expectations . . . . .	40
2.6 Evolutionary Changes in Expectation of Life . . . . .	43
2.7 An Evolutionary Process Governing Survivorship . . . . .	47
2.8 Historical Attempts at Modeling Survivorship . . . . .	52
2.9 Modeling a Force of Mortality for the Whole of Life . . . . .	55
2.10 Computer Experiments in Fitting Survivorship Models to Swedish Historical Data . . . . .	59
2.11 Heterogeneity in Survivorship . . . . .	65

2.12 Further Reading . . . . . 68  
 Problems and Miscellaneous Complements . . . . . 69  
 References . . . . . 74

**Chapter 3. Theories of Competing Risks and Multiple Decrement Life**

**Tables** . . . . . 76

3.1 Introduction . . . . . 76  
 3.2 Mortality in a Cohort with Competing Risks of Death . . . . . 77  
 3.3 Models of Competing Risks Based on Latent Life Spans . . . . . 79  
 3.4 Simple Parametric Models of Competing Risks . . . . . 81  
 Examples: 3.4.1 Constant Forces of Mortality 81. – 3.4.2 A Survival Function on  $R_+^3$   
 82  
 3.5 Equivalent Models of Competing Risks . . . . . 82  
 3.6 Eliminating Causes of Death and Nonidentifiability . . . . . 84  
 Examples: 3.6.1 A Case where the Functions  $S(3, x)$  and  $S^0(3, x)$  Differ 86. – 3.6.2 A  
 Graphical Example Comparing the Survival Functions  $S(x)$ ,  $S(3, x)$ , and  $S^0(3, x)$  86  
 3.7 Estimating a Multiple Decrement Life Table from Period Data . . . 87  
 3.8 Estimating Single Decrement Life Tables from Multiple Decrement  
 Life Tables . . . . . 90  
 3.9 Evolutionary Changes in the Structure of Causes of Death . . . . . 93  
 3.10 Graphs of Multiple Decrement Life Tables – A Study of  
 Proportional Forces of Mortality . . . . . 95  
 3.11 Graphs of Single Decrement Life Tables Associated with Multiple  
 Decrement Tables . . . . . 99  
 3.12 Graphs of Latent Survival Functions and Forces of Mortality . . . 101  
 3.13 An Evolutionary Model of Competing Risks . . . . . 104  
 Problems and Miscellaneous Complements . . . . . 106  
 References . . . . . 111

**Chapter 4. Models of Maternity Histories and Age-Specific Birth Rates** . 112

4.1 Introduction . . . . . 112  
 4.2 A Potential Birth Process . . . . . 113  
 4.3 Cohort Net and Gross Maternity Functions . . . . . 116  
 4.4 Parity Progression Ratios . . . . . 119  
 4.5 Parametric Distributions of Waiting Times Among Live Births . . 121  
 Examples: 4.5.1 Applications of the Exponential Distribution 122. – 4.5.2 A Sim-  
 plified Model Based on the Exponential Distribution 123. – 4.5.3 A Double Ex-  
 ponential Distribution 124. – 4.5.4 Distributions Based on Risk Functions 126  
 4.6 Parametric Forms of the Distribution of Age at First Marriage  
 in a Cohort . . . . . 128  
 Examples: 4.6.1 A Model Based on a Double Exponential Risk Function 128. – 4.6.2  
 A Model Based on the Lognormal Distribution 129. – 4.6.3 Validation of Lognormal

130. – 4.6.4 On the Joint Distribution of the Ages of Brides and Grooms – The Bivariate Lognormal 133

4.7 Heterogeneity in Waiting Times Among Live Births . . . . . 136

Examples: 4.7.1 Gamma Mixtures of Gamma Distributions 137. – 4.7.2 Variances, Covariances, and Correlations of Waiting Times Among Live Births 140. – 4.7.3 Conditional Distributions of the Random Variable  $A$  141. – 4.7.4 Distribution of Waiting Times to  $n$ -th Live Birth 142

4.8 An Age-Dependent Potential Birth Process . . . . . 144

Example: 4.8.1 Maternity Histories in a Nineteenth Century Belgian Commune – La Hulpe 149

4.9 An Evolutionary Potential Birth Process . . . . . 153

4.10 The Evolution of Period Fertility in Sweden – 1780 to 1975 . . . . . 159

4.11 Further Reading . . . . . 163

Problems and Miscellaneous Complements . . . . . 164

References . . . . . 174

**Chapter 5. A Computer Software Design Implementing Models of Maternity Histories . . . . . 177**

5.1 Introduction . . . . . 177

5.2 Semi-Markov Processes in Discrete Time with Stationary Transition Probabilities . . . . . 177

5.3 A Decomposition of Birth Intervals . . . . . 182

5.4 On Choosing Component Functions of the Model . . . . . 187

5.5 An Overview of MATHIST – A Computer Simulation System . . 194

5.6 Applications of MATHIST – Two Simulation Runs in Class One . 201

5.7 A Factorial Experiment Based on Class Two Runs in MATHIST . . 204

5.7.1 An Overview of Computer Input . . . . . 205

5.7.2 Phenomenological and Population Policy Implications of Simulated Cohort Total Fertility Rates and Their Variances . 210

5.7.2.1 Phenomenological Implications . . . . . 211

5.7.2.2 Implications for Population Policy . . . . . 212

5.7.3 Comparisons of Simulated Cohort and Period Age – Specific Fertility Rates . . . . . 213

5.7.3.1 Comparisons of Total Fertility Rates . . . . . 214

5.7.3.2 Comparisons of Birth Rates for the Age Group [15, 20) . . . . . 214

5.7.3.3 Comparisons of Age Groups with Maximum Birth Rates . . . . . 215

5.7.3.4 On the Plausibility of the Mathematical Assumptions Underlying MATHIST . . . . . 216

5.7.4 Computer Generated Graphs of Selected Output from MATHIST . . . . . 219

5.8 A Stochastic Model of Anovulatory Sterile Periods Following Live Births . . . . . 221

Example: 5.8.1 Numerical Examples Based on a Parametric Model 225

5.9 A Semi-Markovian Model for Waiting Times to Conception Under  
Contraception . . . . . 228

Examples: 5.9.1 A One-Step Transition Matrix of Density Functions for Spacers 231. .  
– 5.9.2 A One-Step Transition Matrix of Density Functions for Limiters 232

5.10 Notes on Cohort and Period Projections of Fertility . . . . . 233

5.11 Further Reading . . . . . 234

Problems and Miscellaneous Complements . . . . . 235

References . . . . . 241

**Chapter 6. Age-Dependent Models of Maternity Histories Based on Data  
Analyses . . . . . 243**

6.1 Introduction . . . . . 243

6.2 Age-Dependent Semi-Markov Processes in Discrete Time with  
Stationary Transition Probabilities . . . . . 244

6.3 An Age-Dependent Semi-Markovian Model of Maternity Histories . 248

6.4 On Choosing Computer Input for an Age-Dependent Model of  
Maternity Histories . . . . . 256

6.5 Estimates of Fecundability Functions Based on Null Segments and  
Other Computer Input . . . . . 261

6.6 Numerical Specifications of Four Computer Runs with Inputs Based  
on Survey Data . . . . . 266

6.7 Computer Output Based on Survey Data . . . . . 269

6.8 Further Assessment of the Quality of Calculations in Sect. 6.7 and  
Conclusions . . . . . 276

6.9 A Non-Markovian Model for the Taichung Medical IUD  
Experiment . . . . . 280

6.10 Estimates of Transition Functions Associated with First IUD  
Segment in Taichung Model . . . . . 285

6.11 Validation of Taichung Model . . . . . 288

6.12 State and Fertility Profiles for Taichung Limiters . . . . . 293

6.13 Implications of the Taichung Experiment for Evaluating Family  
Planning Programs . . . . . 298

6.14 On Measuring the Fertility Impact of Family Planning Programs . 299

6.15 Conclusions and Further Reading . . . . . 302

Problems and Miscellaneous Complements . . . . . 304

References . . . . . 307

**Chapter 7. Population Projection Methodology Based on Stochastic  
Population Processes . . . . . 309**

7.1 Introduction . . . . . 309

7.2 Basic Functions Underlying a Branching Process . . . . . 310

7.3	Basic Random Functions and Their Means . . . . .	312
7.4	Explicit Formulas for Mean Functions . . . . .	315
7.5	Leslie Matrix Type Recursive Formulas for Mean Functions . . . . .	317
7.6	A Brief Review of Literature . . . . .	320
7.7	Stochastic Variability in Population Structure as a Gaussian Process . . . . .	322
7.8	A Representation of Population Structure Based on Birth Cohorts . . . . .	326
7.9	Covariance Functions for the Birth Process and Live Individuals . . . . .	328
7.10	Product Moments of the Actual and Potential Birth Processes . . . . .	334
7.11	Product Moment Functions as Solutions of Renewal Equations . . . . .	340
7.12	Asymptotic Formulas for Mean and Covariance Functions in the Time Homogeneous Case . . . . .	343
7.13	Period Demographic Indicators in Populations with Time Inhomogeneous Laws of Evolution . . . . .	347
7.14	Asymptotic Formulas for Period Demographic Indicators in the Time Homogeneous Case . . . . .	351
7.15	A Female Dominant Two-Sex Population Process . . . . .	354
7.16	An Overview of a Computer Software Design Implementing Population Projection Systems . . . . .	358
7.17	Four Computer Runs in the Time Homogeneous Case – A Study of Population Momentum . . . . .	363
7.17.1	Guidelines for Interpreting Graphs of Period Mean Total Population . . . . .	367
7.17.2	Guidelines for Interpreting Graphs of Period Rates of Population Growth, Crude Birth Rates, and Crude Death Rates . . . . .	367
7.17.3	Guidelines for Interpreting Graphs of Distances of Period Age Distributions from Their Asymptotic Stable Forms . . . . .	368
7.17.4	Implications for Population Policy . . . . .	371
7.18	A Study of Changing Mortality and Constant Fertility in the Time Inhomogeneous Case . . . . .	371
7.18.1	Guidelines for Interpreting Mean Total Population and Mean Total Births and Deaths . . . . .	373
7.18.2	Guidelines for Interpreting Period Crude Birth and Death Rates and Rates of Population Growth . . . . .	375
7.18.3	Guidelines for Interpreting Period Age Densities in the Time Inhomogeneous Run . . . . .	376
7.19	Further Reading . . . . .	376
	Problems and Miscellaneous Complements . . . . .	377
	References . . . . .	383
	<b>Author Index . . . . .</b>	<b>385</b>
	<b>Subject Index . . . . .</b>	<b>387</b>