

Contents Volume 1

Introduction	1
Notes and Remarks	16
C*-Algebras and von Neumann Algebras	17
2.1. C*-Algebras	19
2.1.1. Basic Definitions and Structure	19
2.2. Functional and Spectral Analysis	25
2.2.1. Resolvents, Spectra, and Spectral Radius	25
2.2.2. Positive Elements	32
2.2.3. Approximate Identities and Quotient Algebras	39
2.3. Representations and States	42
2.3.1. Representations	42
2.3.2. States	48
2.3.3. Construction of Representations	54
2.3.4. Existence of Representations	58
2.3.5. Commutative C*-Algebras	61
	vii

2.4. von Neumann Algebras	65
2.4.1. Topologies on $\mathcal{L}(\mathfrak{H})$	65
2.4.2. Definition and Elementary Properties of von Neumann Algebras	71
2.4.3. Normal States and the Predual	75
2.4.4. Quasi-Equivalence of Representations	79
2.5. Tomita–Takesaki Modular Theory and Standard Forms of von Neumann Algebras	83
2.5.1. σ -Finite von Neumann Algebras	84
2.5.2. The Modular Group	86
2.5.3. Integration and Analytic Elements for One-Parameter Groups of Isometries on Banach Spaces	97
2.5.4. Self-Dual Cones and Standard Forms	102
2.6. Quasi-Local Algebras	118
2.6.1. Cluster Properties	118
2.6.2. Topological Properties	129
2.6.3. Algebraic Properties	133
2.7. Miscellaneous Results and Structure	136
2.7.1. Dynamical Systems and Crossed Products	136
2.7.2. Tensor Products of Operator Algebras	142
2.7.3. Weights on Operator Algebras; Self-Dual Cones of General von Neumann Algebras; Duality and Classification of Factors; Classification of C^* -Algebras	145
Notes and Remarks	152
Groups, Semigroups, and Generators	157
3.1. Banach Space Theory	159
3.1.1. Uniform Continuity	161
3.1.2. Strong, Weak, and Weak* Continuity	163
3.1.3. Convergence Properties	183
3.1.4. Perturbation Theory	189
3.1.5. Approximation Theory	198
3.2. Algebraic Theory	205
3.2.1. Positive Linear Maps and Jordan Morphisms	205
3.2.2. General Properties of Derivations	228

3.2.3. Spectral Theory and Bounded Derivations	244
3.2.4. Derivations and Automorphism Groups	259
3.2.5. Spatial Derivations and Invariant States	263
3.2.6. Approximation Theory for Automorphism Groups	285
Notes and Remarks	298
Decomposition Theory	309
4.1. General Theory	311
4.1.1. Introduction	311
4.1.2. Barycentric Decompositions	315
4.1.3. Orthogonal Measures	333
4.1.4. Borel Structure of States	344
4.2. Extremal, Central, and Subcentral Decompositions	353
4.2.1. Extremal Decompositions	353
4.2.2. Central and Subcentral Decompositions	362
4.3. Invariant States	367
4.3.1. Ergodic Decompositions	367
4.3.2. Ergodic States	386
4.3.3. Locally Compact Abelian Groups	400
4.3.4. Broken Symmetry	416
4.4. Spatial Decomposition	432
4.4.1. General Theory	433
4.4.2. Spatial Decomposition and Decomposition of States	442
Notes and Remarks	451
References	459
Books and Monographs	461
Articles	464
List of Symbols	481
Subject Index	487