

Contents

Introduction	xi
1. The political context	1
1.1 Introduction	1
1.2 What part does history of mathematics currently occupy in national curricula?	2
1.2.1 Argentina	2
1.2.2 Austria	3
1.2.3 Brazil	3
1.2.4 China	4
1.2.5 Denmark	5
1.2.6 France	7
1.2.7 Greece	8
1.2.8 Israel	9
1.2.9 Italy.....	9
1.2.10 Japan.....	10
1.2.11 Netherlands.....	11
1.2.12 New Zealand.....	12
1.2.13 Norway	14
1.2.14 Poland	15
1.2.15 United Kingdom	16
1.2.16 United States of America	18
1.3 History of mathematics in curricula and schoolbooks: a case study of Poland	19
1.3.1 History of mathematics in mathematics curricula	20
1.3.2 History of mathematics in mathematics school-books	21
1.3.3 Final remarks	28
1.4 Policy and politics in the advocacy of a historical component	29
1.4.1 Political authorities (at all levels).....	31
1.4.2 Teacher associations	31
1.4.3 Professional mathematics associations	32
1.4.4 Tertiary teachers	32
1.4.5 Parents	33
1.4.6 Textbook authors	33
1.5 Quotations on the use of history of mathematics in mathematics teaching and learning	33

2. Philosophical, multicultural and interdisciplinary issues	39
.....	
2.1 Introduction	39
2.2 Philosophical issues	40
2.2.1 Historical investigation, evidence and interpretation	40
2.2.2 Philosophy of mathematics, old and new	42
2.2.3 The ends of the spectrum	44
2.3 Multicultural issues	46
2.3.1 Introduction	46
2.3.2 Multiculturalism inside the history of mathematics	47
2.3.3 Mathematics as a human enterprise	50
2.4 Interdisciplinary issues	52
2.4.1 Introduction	52
2.4.2 History of mathematics and the study of history	53
2.4.3 History of mathematics linking topics within mathematics	53
2.4.4 History of mathematics linking mathematics with other disciplines	55
2.5 Conclusion	61
3. Integrating history: research perspectives	63
.....	
3.1 Introduction	63
3.2 The historical dimension: from teacher to learner	66
3.3 The indirect genetic approach to calculus	71
3.4 Stochastics teaching and cognitive development	74
3.5 Ancient problems for the development of strategic thinking	78
3.6 Difficulties with series in history and in the classroom	82
3.7 On potentialities, limits and risks	86
3.8 Suggestions for future research	90
4. History of Mathematics for Trainee Teachers	91
.....	
4.1 Earlier views on history in teacher education	91
4.2 International overview	93
4.3 Examples of current practice	110
4.3.1 Current practice in initial teacher training	110
4.3.2 Current practice in in-service training	131
4.4 Issues of Concern	140

5. Historical formation and student understanding of mathematics.....	143
5.1 Introduction	143
5.2 The role of historical analysis in predicting and interpreting students' difficulties in mathematics	149
5.3 The relevance of historical studies in designing and analysing classroom activities.....	154
5.3.1 Bringing historical texts into the classroom: the 'voices and echoes' games.....	155
5.3.2 Indirect use of historical and epistemological studies in the design of activities for students.....	156
5.3.3 The example of linear algebra.....	157
5.3.4 The example of calculus	159
5.3.5 Research on the methodology of history-based design of activities for students	160
5.4 Epistemological assumptions framing interpretations of students understanding of mathematics	162
5.4.1 The 'epistemological obstacles' perspective.....	162
5.4.2 A socio-cultural perspective	163
5.4.3 The 'voices and echoes' perspective.....	165
5.5 Conclusions: guidelines and suggestions for future research	168
6. History in support of diverse educational requirements — opportunities for change.....	171
6.1 Introduction	171
6.2 Educational, cultural, social and economic diversity in primary, secondary and tertiary settings.....	172
6.2.1 Primary education and the use of mathematics history in the classroom	172
6.2.2 Under-served (limited resources) students.....	174
6.2.3 Alternative educational pathways: adult learners returning to mathematics education, vocational education and training	179
6.2.4 Minority school populations	184
6.2.5 Students having educational challenges.....	187
6.2.6 Mathematically gifted and talented students	188
6.3 Opportunities for change	195
6.3.1 Teacher education	195
6.4 Conclusion	199
7. Integrating history of mathematics in the classroom:an analytic survey.....	201
7.1 Introduction	201
7.2 Why should history of mathematics be integrated in mathematics education?	202

7.3	How may history of mathematics be integrated in mathematics education?	208
7.3.1	Direct historical information.....	208
7.3.2	A teaching approach inspired by history.....	208
7.3.3	Mathematical awareness	211
7.4	Ideas and examples for classroom implementation	213
7.4.1	Historical Snippets.....	214
7.4.2	Student research projects based on history texts	215
7.4.3	Primary Sources.....	216
7.4.4	Worksheets	216
7.4.5	Historical packages.....	217
7.4.6	Taking advantage of errors, alternative conceptions, change of perspective, revision of implicit assumptions, intuitive arguments etc.....	219
7.4.7	Historical problems.....	224
7.4.8	Mechanical instruments	227
7.4.9	Experiential mathematical activities	228
7.4.10	Plays	229
7.4.11	Films and other visual means.....	230
7.4.12	Outdoor experiences	231
7.4.13	The WWW.....	232

8. Historical support for particular subjects

.....241

8.1	Introduction	241
8.2	Teaching projects inspired by history.....	243
8.2.1	Examples from algebra and analysis.....	243
8.2.2	A heuristic introduction to analysis implicitly inspired by its historical development	245
8.2.3	How may history help the teaching of probabilistic concepts?.....	248
8.2.4	Trigonometry in the historical order	252
8.3	Cultural aspects of mathematics in a historical perspective	253
8.3.1	Number systems and their representations.....	253
8.3.2	The Pythagorean theorem in different cultures	258
8.3.3	Measuring distances: Heron vs. Liu Hui.....	262
8.4	Detailed treatment of particular examples	264
8.4.1	Introducing complex numbers: an experiment.....	264
8.4.2	Intertwining a mathematical topic with other (non-) mathematical topics	265
8.4.3	Surveyors' problems	273
8.4.4	Theory of proportion and the geometry of areas.....	276
8.4.5	Deductive vs intuitive thinking: an example from the calculus	279
8.4.6	Tracing the root of the abstract concept of a set.....	281
8.4.7	Discrete mathematics: an example.....	282
8.4.8	The relation between geometry and physics: an example	283

8.5	Improving mathematical awareness through the history of mathematics.....	286
8.5.1	History of mathematics education.....	286
8.5.2	Teaching secondary mathematics in a historical perspective	288
8.5.3	Adults' mathematics educational histories.....	289
 9. The use of original sources in the mathematics classroom		
.....		291
9.1	Introduction	291
9.2	Motivations, aims and uses	292
9.2.1	The specific value and quality of primary sources	292
9.2.2	Understanding the evolution of ideas.....	294
9.2.3	Experiencing the relativity of truth and the human dimension of mathematical activity... ..	295
9.2.4	Relations between mathematics and philosophy.....	296
9.2.5	Simplicity, motivation and didactics.....	297
9.2.6	Perspectives on mathematics education	297
9.2.7	Local Mathematics.....	298
9.3	Sources, hermeneutics and language	298
9.4	Integrating original sources in pre-service teacher education	299
9.4.1	Example 1: Egyptian measures of angles.....	299
9.4.2	Example 2: complex numbers in geometry and algebra	303
9.5	Integrating Original Sources in the Classroom.....	307
9.5.1	Example 1: Greek surveying: the tunnel of Samos	307
9.5.2	Example 2: An 18th century treatise on conic sections.....	310
9.6	Didactical strategies for integrating sources	313
9.6.1	The triad: text - context - reader.....	313
9.6.2	Classroom strategies	313
9.7	Evaluation, research questions and issues of concern	316
 10. Non-standard media and other resources		
.....		329
10.1	Introduction	329
10.1.1	Why other media?	329
10.1.2	And which media?	330
10.1.3	Affect and effect	331
10.1.4	Media and cognitive aspects of learning.....	333
10.1.5	Media and assessment.....	334
10.2	Learning through history and non-standard media	335
10.2.1	Mathematical Dramatisation.....	335
10.2.2	Ancient instruments in the modern classroom	343

10.2.3	Inquiring mathematics with history and software	351
10.3	Resources for history of mathematics on the World Wide Web.....	358
10.3.1	Teachers, learners and the World Wide Web.....	358
10.3.2	Web historical resources for the mathematics teacher	362

11. Bibliography for further work in the area

	371
11.1	Introduction	371
11.2	Chinese.....	373
11.3	Danish	383
11.4	Dutch.....	386
11.5	English	389
11.6	French.....	404
11.7	German	405
11.8	Greek	411
11.9	Italian.....	414
11.10	Collections of articles (special issues).....	416
11.10.1	Journals (special issues).....	416
11.10.2	Books.....	417
	Notes on contributors	419
	Index.....	429