

Contents

Foreword	vii
Introduction	1
Preview	13
A recipe	13
Contents	31
Notation	36
I Point Processes	41
1 An intuitive approach	41
1.1 A brief shower	41
1.2 Sample cloud mixtures	43
1.3 Random sets and random measures	44
1.4 The mean measure	45
1.5* Enumerating the points	46
1.6 Definitions	47
2 Poisson point processes	48
2.1 Poisson mixtures of sample clouds	48
2.2 The distribution of a point process	49
2.3 Definition of the Poisson point process	50
2.4 Variance and covariance	51
2.5* The bivariate mean measure	52
2.6 Lévy processes	54
2.7 Superpositions of zero-one point processes	56
2.8 Mappings	58
2.9* Inverse maps	58
2.10* Marked point processes	62
3 The distribution	63
3.1 Introduction	63
3.2* The Laplace transform	64
3.3 The distribution	65
3.4* The distribution of simple point processes	67
4 Convergence	69
4.1 Introduction	69
4.2 The state space	70
4.3 Weak convergence of probability measures on metric spaces	72

Starred sections may be skipped on a first reading.

4.4	Radon measures and vague convergence	76
4.5	Convergence of point processes	78
5	Converging sample clouds	81
5.1	Introduction	81
5.2	Convergence of convex hulls, an example	83
5.3	Halfspaces, convex sets and cones	84
5.4	The intrusion cone	87
5.5	The convergence cone	89
5.6*	The support function	92
5.7	Almost-sure convergence of the convex hulls	93
5.8	Convergence to the mean measure	96
II	Maxima	100
6	The univariate theory: maxima and exceedances	100
6.1	Maxima	100
6.2	Exceedances	101
6.3	The domain of the exponential law	101
6.4	The Poisson point process associated with the limit law	102
6.5*	Monotone transformations	104
6.6*	The von Mises condition	105
6.7*	Self-neglecting functions	108
7	Componentwise maxima	110
7.1	Max-id vectors	111
7.2	Max-stable vectors, the stability relations	112
7.3	Max-stable vectors, dependence	114
7.4	Max-stable distributions with exponential marginals on $(-\infty, 0)$	117
7.5*	Max-stable distributions under monotone transformations	119
7.6	Componentwise maxima and copulas	121
III	High Risk Limit Laws	123
8	High risk scenarios	123
8.1	Introduction	123
8.2	The limit relation	125
8.3	The multivariate Gaussian distribution	126
8.4	The uniform distribution on a ball	128
8.5	Heavy tails, returns and volatility in the DAX	130
8.6	Some basic theory	131
9	The Gauss-exponential domain, rotund sets	135
9.1	Introduction	136

9.2	Rotund sets	138
9.3	Initial transformations	140
9.4	Convergence of the quotients	143
9.5	Global behaviour of the sample cloud	146
10	The Gauss-exponential domain, unimodal distributions	147
10.1	Unimodality	147
10.2*	Caps	149
10.3*	L^1 -convergence of densities	152
10.4	Conclusion	154
11	Flat functions and flat measures	156
11.1	Flat functions	156
11.2	Multivariate slow variation	157
11.3	Integrability	159
11.4*	The geometry	160
11.5	Excess functions	166
11.6*	Flat measures	167
12	Heavy tails and bounded vectors	170
12.1	Heavy tails	170
12.2	Bounded limit vectors	173
13	The multivariate GPDs	176
13.1	A continuous family of limit laws	176
13.2	Spherical distributions	178
13.3	The excess measures and their symmetries	179
13.4	Projection	180
13.5	Independence and spherical symmetry	180
IV	Thresholds	182
14	Exceedances over horizontal thresholds	183
14.1	Introduction	183
14.2	Convergence of the vertical component	185
14.3*	A functional relation for the limit law	186
14.4*	Tail self-similar distributions	187
14.5*	Domains of attraction	190
14.6	The Extension Theorem	192
14.7	Symmetries	193
14.8	The Representation Theorem	195
14.9	The generators in dimension $d = 3$ and densities	196
14.10	Projections	198
14.11	Sturdy measures and steady distributions	200
14.12	Spectral stability	203
14.13	Excess measures for horizontal thresholds	204

14.14	Normalizing curves and typical distributions	205
14.15	Approximation by typical distributions	209
15	Horizontal thresholds – examples	211
15.1	Domains for exceedances over horizontal thresholds	211
15.2	Vertical translations	211
15.3	Cones and vertices	218
15.4	Cones and heavy tails	222
15.5*	Regular variation for matrices in \mathcal{A}^h	227
16	Heavy tails and elliptic thresholds	230
16.1	Introduction	230
16.2	The excess measure	235
16.3	Domains of elliptic attraction	240
16.4	Convex hulls and convergence	243
16.5	Typical densities	245
16.6	Roughening and vague convergence	247
16.7	A characterization	251
16.8*	Interpolation of ellipsoids, and twisting	256
16.9	Spectral decomposition, the basic result	258
17	Heavy tails – examples	263
17.1	Scalar normalization	264
17.2	Scalar symmetries	268
17.3*	Coordinate boxes	273
17.4	Heavy and heavier tails	275
17.5*	Maximal symmetry	278
17.6*	Stable distributions and processes	282
17.7*	Elliptic thresholds	285
18	Regular variation and excess measures	295
18.1	Regular variation	295
18.2	Discrete skeletons	299
18.3*	Regular variation in \mathcal{A}^+	300
18.4	The Meerschaert spectral decomposition	304
18.5	Limit theory with regular variation	312
18.6	Symmetries	314
18.7*	Invariant sets and hyperplanes	316
18.8	Excess measures on the plane	318
18.9	Orbits	320
18.10*	Uniqueness of extensions	326
18.11*	Local symmetries	329
18.12	Jordan form and spectral decompositions	333
18.13	Lie groups and Lie algebras	336
18.14	An example	344

V	Open problems	348
19	The stochastic model	349
20	The statistical analysis	356
	Bibliography	361
	Index	369