

CONTENTS

VOLUME II

Preface to Volume II	xvii
List of symbols and notation used in Volume II	xix

17. Ordinary Differential Equations

By KAREL REKTORYS

Introductory remark	1
17.1. Classification of differential equations. Ordinary and partial differential equations. Order of a differential equation. Systems of differential equations	2
17.2. Basic concepts. Solution (integral) of a differential equation. Theorems relating to existence and uniqueness of solution. General integral, particular integral, singular integral	2
17.3. Elementary methods of integration of equations of the first order. Separation of variables. Homogeneous equations. Linear equations. Bernoulli's equation. Riccati's equation	12
17.4. Exact differential equations. The integrating factor. Singular points	23
17.5. Equations of the first order not solved with respect to the derivative. Lagrange's equation. Clairaut's equation. Singular solutions	27
17.6. Orthogonal and isogonal (oblique) trajectories	35
17.7. Differential equations of order n . Simple types of equations of order n . The method of a parameter	36
17.8. First integral of a differential equation of the second order. Reduction of the order of a differential equation. Equations, the left-hand sides of which are exact derivatives	41
17.9. Dependence of solutions on parameters of the differential equation and on initial conditions	45
17.10. Asymptotic behaviour of integrals of differential equations (for $x \rightarrow +\infty$). Oscillatory solutions. Periodic solutions	46
17.11. Linear equations of the n -th order	50
17.12. Non-homogeneous linear equations. The method of variation of parameters	55
17.13. Homogeneous linear equations with constant coefficients. Euler's equation	57
17.14. Non-homogeneous linear equations with constant coefficients and a special right-hand side	62
17.15. Linear equations of the second order with variable coefficients. Transformation into self-adjoint form, into normal form. Invariant. Equations with regular singularities (equations of the Fuchsian type). Some special equations (Bessel's equation etc.)	66
17.16. Discontinuous solutions of linear equations	75
17.17. Boundary value problems. Eigenvalue problems. Expansion theorem. Green's function	78
17.18. Systems of ordinary differential equations	99

17.19.	Dependence of solutions of systems of differential equations on initial conditions and on parameters of the system. Stability of solutions	112
17.20.	First integrals of a system of differential equations	116
17.21.	Table of solved differential equations	120
	(a) Equations of the first order	121
	(b) Linear equations of the second order.	131
	(c) Linear equations of higher orders. Nonlinear equations. Systems.	141

18. Partial Differential Equations

By KAREL REKTORYS

	Informative remark	147
18.1.	Partial differential equations in general. Basic concepts. Questions concerning the concept of general solution. Cauchy's problem, boundary value problems, mixed problems. Kovalewski's theorem. Characteristics. Well-posed problems.	148
18.2.	Partial differential equations of the first order. Homogeneous and nonhomogeneous linear equations, nonlinear equations. Complete, general and singular integrals. Solution of the Cauchy problem	156
18.3.	Linear equations of the second order. Classification	172
18.4.	Elliptic equations. The Laplace equation, the Poisson equation. The Dirichlet and Neumann problems. Properties of harmonic functions. The fundamental solution. Green's function. Potentials	174
18.5.	Hyperbolic equations. Wave equation, the Cauchy problem, the mixed problem. Generalized solutions of hyperbolic equations	191
18.6.	Parabolic equations. The heat-conduction equation. The Cauchy problem. Mixed boundary value problems	197
18.7.	Some other problems of partial differential equations. Systems of equations. Pfaffian equation. Equations of higher order, biharmonic equation. Potential flow, the Navier-Stokes equations	201
18.8.	Elliptic boundary value problems of arbitrary order. Generalized solutions. Eigenvalue problems	204
18.9.	Weak solutions of boundary value problems. Nonlinear problems	207
18.10.	Application of variational methods to the solution of partial differential equations containing time. The method of discretization in time (the Rothe method, the "horizontal" method of lines)	215

19. Integral Equations

By KAREL REKTORYS

19.1.	Integral equations of Fredholm's type. Solvability, Fredholm's theorems. Systems of integral equations	220
19.2.	Equations with degenerate kernels	228
19.3.	Equations with symmetric kernels	231
19.4.	The resolvent	234
19.5.	Equations involving weak singularities. Singular equations	238
19.6.	Equations of Volterra type	240
19.7.	Integral equations of the first kind	241

20. Functions of One and More Complex Variables

A. *Functions of One Complex Variable*

By KAREL REKTORYS

20.1.	Fundamental concepts. Limit and continuity. The derivative. The Cauchy-Riemann equations. Applications of the theory of functions of one complex variable.	243
20.2.	Integral of a function of a complex variable. The Cauchy integral theorem. Cauchy's integral formula	249
20.3.	Integrals of Cauchy's type. The Plemelj formulae	254
20.4.	Series. Taylor's series, Laurent's series. Singular points of holomorphic functions	259
20.5.	The residue of a function. The residue theorem and its applications	269
20.6.	Logarithm, power. Analytic continuation. Analytic functions	272

B. *Functions of Several Complex Variables*

By JAROSLAV FUKA

	Introductory remark	277
20.7.	Important regions in \mathbb{C}^n	278
20.8.	Functions of several complex variables. Complex derivative, complex differential, holomorphic functions	282
20.9.	Cauchy-Riemann equations. Pluriharmonic functions	283
20.10.	Local properties of holomorphic functions. The Cauchy integral formula. The Taylor expansion	284
20.11.	On some different properties of functions of one and several complex variables. Analytic continuation. Domain of holomorphy. Biholomorphic mapping	286

21. Conformal Mapping

By JAROSLAV FUKA

21.1.	The concept of conformal mapping	289
21.2.	Existence and uniqueness of conformal mapping	293
21.3.	Methods of performing conformal mappings	296
21.4.	Boundary properties of conformal mappings	304

Numerical Methods in Conformal Mappings

21.5.	Variational methods	305
21.6.	The method of integral equations	308
21.7.	Mapping of "adjacent" regions	310
21.8.	Mapping of the upper half-plane on a polygon	311
21.9.	A small dictionary of conformal mappings	312

22. Fundamentals of the Theory of Sets and Functional Analysis

By KAREL REKTORYS

22.1.	Open and closed sets of points in E_n . Regions	319
22.2.	Metric spaces	323
22.3.	Complete, separable, compact spaces	327
22.4.	Linear spaces. Normed spaces. Banach and Hilbert spaces. Orthogonal systems. Generalized derivatives, Sobolev spaces, embedding theorems. Distributions	330
22.5.	Linear and other operators in metric spaces. Banach's theorem on contraction mapping. Functionals. Adjoint operators, adjoint (dual) spaces. Completely continuous operators.	344
22.6.	Operators and operator equations in Hilbert spaces	352
	(a) Bounded operators	352
	(b) Unbounded operators	358
22.7.	Abstract functions. The Bochner integral	364
22.8.	The Gâteaux differential and related concepts	367

23. Calculus of Variations

By FRANTIŠEK NOŽIČKA

A. Problems of the First Category (*Elementary Problems of the Calculus of Variations*)

23.1.	Curves of the r -th class, distance of order r between two curves, ε -neighbourhood of order r of a curve	374
23.2.	Extrema of functionals of the form $\int_a^b F(x, y, y') dx$	376
23.3.	Variation of a function and variation of the functional I	378
23.4.	Necessary condition for an extremum of the functional I	381
23.5.	Special cases of the Euler equation. The brachistochrone problem	381

B. Problems of the Second Category (*Extrema of Functionals of the Form $\int_a^b F(x, y_1, \dots, y_n, y'_1, \dots, y'_n) dx$*)

23.6.	Some concepts and definitions	385
23.7.	Formulation of the variational problem	386
23.8.	Necessary conditions for an extremum of the functional I	386

C. Problems of the Third Category (*Extrema of Functionals of the Form $\int_a^b F(x, y, y', \dots, y^{(n)}) dx$*)

23.9.	Formulation of the problem	388
23.10.	A necessary condition for the extremum of the functional (23.9.1)	389
23.11.	Generalization to the case of an arbitrary finite number of functions sought	391

D. Problems of the Fourth Category (*Functionals Depending on a Function of n Variables*)

23.12.	Some concepts and definitions	392
23.13.	Formulation of the variational problem and necessary conditions for an extremum	394

*E. Problems of the Fifth Category (Variational Problems
with "Moving (Free) Ends of Admissible Curves")*

23.14.	Formulation of the simplest problem	395
23.15.	Necessary conditions for an extremum	396

F. Problems of the Sixth Category (the Isoperimetric Problem in the Simplest Case)

23.16.	Formulation of the problem	399
23.17.	A necessary condition for an extremum	399

G. Problems of the Seventh Category (Parametric Variational Problems)

23.18.	Formulation of the problem	403
23.19.	Necessary conditions for an extremum of the functional I	404

H. Problems of the Eighth Category (Variational Problems with Constraints)

23.20.	Formulation of the variational problem and necessary conditions for an extremum	405
23.21.	Variational problems with generalized constraints	406
23.22.	Canonical form of the Euler equations. Hamiltonian equations	407

**24. Variational Methods for Numerical Solution of Boundary-Value Problems for
Differential Equations. Finite Element Method. Boundary Element Method**

By MILAN PRÁGER

24.1.	Introduction. Theoretical background. Table of boundary-value problems	409
24.2.	Fundamental approximation methods	422
	(a) The Ritz method	422
	(b) The Galerkin method	427
24.3.	The finite element method	428
	(a) Decompositions and finite elements	428
	(α) One-dimensional finite elements	431
	(β) Two-dimensional finite elements	433
	(γ) Three-dimensional finite elements	441
	(b) The finite element spaces	443
	(c) Convergence of the finite element method	447
24.4.	Computational aspects of the finite element method	450
24.5.	Computation of eigenvalues and eigenfunctions by the finite element method	456
24.6.	Variational methods for numerical solution of parabolic equations	463
24.7.	The boundary element method	469

25. Approximate Solution of Ordinary Differential Equations

By EMIL VITÁSEK

25.1.	Introduction	478
-------	------------------------	-----

A. Initial Value Problems

25.2.	Euler's method. Error estimates	483
-------	---	-----

25.3.	General one-step method	489
	(a) Taylor's expansion methods	491
	(b) Runge-Kutta methods	492
25.4.	Linear k -step method	495
	(a) Methods of numerical integration. Adams-Bashforth method. Adams-Moulton method	502
	(b) Methods of numerical differentiation. Backward difference methods . .	504
25.5.	The use and comparison of Runge-Kutta and linear multistep methods. Predictor-corrector methods	506
25.6.	Extrapolation methods. Richardson's extrapolation, Gragg's method	512

B. Boundary Value Problems

25.7.	Shooting method	515
25.8.	Methods of the transfer and normalized transfer of boundary conditions . .	519
25.9.	Finite difference method	525
25.10.	The eigenvalue problem	528

26. Solution of Partial Differential Equations by Infinite Series (by the Fourier Method)

By KAREL REKTORYS

26.1.	Equation of a vibrating string	534
26.2.	Potential equation and stationary heat-conduction equation	539
26.3.	Heat conduction in rectangular regions	540
26.4.	Heat conduction in an infinite circular cylinder. Application of Bessel functions	542
26.5.	Deflection of a rectangular simply supported plate	543

27. Solution of Partial Differential Equations by the Finite-Difference Method

By EMIL VITÁSEK

27.1.	Basic idea of the finite-difference method	546
27.2.	Principal types of nets	549
	(a) Rectangular nets.	549
	(b) Hexagonal and triangular nets	550
	(c) Polar nets	550
27.3.	Refinement of nets	550
27.4.	Finite-difference formulae for the most frequently occurring operators . .	550
27.5.	Formulation of boundary conditions	551
	(a) Boundary conditions which do not contain derivatives	551
	(b) Boundary conditions containing derivatives	553
27.6.	Error estimates	556
27.7.	Examples. Laplace's equation. Heat-conduction equation. Biharmonic equation	557
27.8.	General scheme of the finite-difference method	562

28. Integral Transforms (Operational Calculus)

By JINDŘICH NEČAS

28.1. One-dimensional infinite transforms (the Laplace, Fourier, Mellin, Hankel transforms)	567
28.2. Applications of the Laplace and Fourier transforms to the solution of differential equations. Examples	570
28.3. Some results of fundamental importance. Tables	574
28.4. Two-dimensional and multidimensional transforms	581
28.5. One-dimensional finite transforms	584

29. Approximate Solution of Fredholm's Integral Equations

By KAREL REKTORYS

29.1. Successive approximations (iterations)	585
29.2. Approximate solution of integral equations making use of quadrature formulae	586
29.3. Replacement of the kernel by a degenerate kernel	589
29.4. The Galerkin method (method of moments) and the Ritz method	589
29.5. Application of the Ritz method to approximate determination of the first characteristic value of an equation with a symmetric kernel	591

30. Numerical Methods in Linear Algebra

By JITKA SEGETHOVÁ AND KAREL SEGETH

A. Solution of Systems of Linear Algebraic Equations

30.1. Gaussian elimination and LU factorization	595
30.2. Computation of the determinant and the inverse matrix	601
30.3. Roundoff error. Iterative improvement of the solution	603
30.4. Singular value decomposition. Solution of systems with singular and rectangular matrices	606
30.5. Sparse systems. Cyclic reduction	611
30.6. Iterative methods. One-point iteration, the Jacobi and Gauss-Seidel methods, successive overrelaxation. Conjugate gradient method	615
30.7. Preconditioned iterative methods. Incomplete factorization	621
30.8. Algebraic multigrid method	625
30.9. Choice of the method. Basic software	626

B. Computation of Eigenvalues and Eigenvectors of Matrices

30.10. Bounds for eigenvalues	628
30.11. Power method	630
30.12. Jacobi method	632
30.13. LR and QR methods	635
30.14. Reducing matrices to simpler forms. The Givens and Householder methods. The Lanczos and Wilkinson methods	639
30.15. Inverse iteration method	644

30.16. Generalized eigenproblem	645
30.17. Choice of the method. Basic software	646

31. Numerical Solution of Algebraic and Transcendental Equations

By MIROSLAV FIEDLER

31.1. Basic properties of algebraic equations	648
31.2. Estimates for the roots of algebraic equations	649
31.3. Connection of roots with eigenvalues of matrices	652
31.4. Some methods for solving algebraic and transcendental equations	652
(a) Method of Bernoulli and Whittaker	653
(b) The Graeffe method and its modifications	654
(c) Newton's method	658
(d) The regula falsi method	658
(e) Bairstow's method	659
(f) The general iterative method	661
31.5. Numerical solution of (nonlinear) systems	662

32. Approximation. Interpolation, Splines

By EMIL VITÁSEK

32.1. The best approximation in a linear normed space	666
32.2. The best approximation in a Hilbert space	667
32.3. The best approximation of continuous functions by polynomials	669
32.4. Jackson's theorems	671
32.5. The Remes algorithm	672
(a) Chebyshev's expansions	674
(b) Economized power series	674
32.6. Polynomial interpolation. Lagrange's interpolation formula. Hermite's interpolation formula	675
32.7. Differences. Interpolation polynomial for equidistant arguments	678
32.8. Trigonometric interpolation	683
32.9. Interpolation by splines	684
(a) Interpolation of the Lagrange type	685
(b) Interpolation of the Hermite type	687

33. Probability Theory

By TOMÁŠ CIPRA

33.1. Random event and probability	688
33.2. Conditional probability and independent events	691
33.3. Random variables and probability distributions	694
33.4. Basic characteristics of random variables	697
33.5. Random vectors	704
33.6. Important discrete distributions	710
33.7. Important continuous distributions	714
33.8. Important multivariate distributions	725

33.9. Transformations of random variables	727
33.10. Some inequalities	729
33.11. Limit theorems in probability theory	730
33.12. Law of large numbers	731
33.13. Central limit theorems	733

34. Mathematical Statistics

By TOMÁŠ CIPRA

34.1. Basic concepts	735
34.2. Sample characteristics	736
34.3. Random sample from normal distribution	738
34.4. Ordered random sample	739
34.5. Elementary statistical treatment	741
34.6. Estimation theory	745
34.7. Point estimators	749
34.8. Interval estimators	752
34.9. Hypothesis testing	755
34.10. Tests of hypotheses on parameters of normal distributions	756
34.11. Goodness of fit tests	760
34.12. Contingency tables	763

35. Topics in Statistical Inference

By TOMÁŠ CIPRA

A. *Regression Analysis. Fitting Curves to Empirical Data. Calculus of Observations*

35.1. Regression in statistics	766
35.2. Linear regression model	768
35.3. Normal linear regression model	773
35.4. Linear regression	775
35.5. Polynomial regression	776
35.6. Generalized linear regression model. Calculus of observations	777
35.7. Nonlinear regression	779

B. *Analysis of Variance*

35.8. Principle of the analysis of variance	782
35.9. One-way classification	783

C. *Multivariate Analysis* 785

D. *Reliability Theory*

35.10. Basic reliability concepts	786
35.11. Estimation of reliability characteristics	789

E. *Statistical Principles of Quality Control*

35.12. Acceptance sampling	792
35.13. Sequential acceptance sampling	795

36. Stochastic Processes

By TOMÁŠ ČIPRA

36.1.	Classification of stochastic processes	797
	<i>A. Markov Processes</i>	
36.2.	Concept of Markov processes	799
36.3.	Examples of Markov processes	802
36.4.	Markov chains	804
	<i>B. Queueing Theory</i>	
36.5.	Service systems	806
36.6.	Examples of service systems	807
	<i>C. Stationary Processes</i>	
36.7.	Correlation properties of stationary processes	810
36.8.	Spectral properties of stationary processes	814

37. Linear Programming

By FRANTIŠEK NOŽIČKA

	Introductory remark	822
37.1.	Formulation of the general problem of linear programming	823
37.2.	Linear optimization problem in normal form	825
37.3.	Linear optimization problem in equality form	827
37.4.	Examples of linear optimization problems solved in practice	828
	(a) Classical transportation problem	828
	(b) Blending problem	829
	(c) Production planning	831
37.5.	Decomposition of a convex polyhedron into its interior and faces	832
37.6.	The set of optimal points of a linear optimization problem	835
37.7.	The concept of feasible basic point	836
37.8.	Exchange of basic variables. Optimality criterion. The degenerate case	839
37.9.	Simplex method. An example	848
37.10.	Finding a feasible basic point	858
37.11.	Duality principle	860
	References	864
	Index	886
	Contents volume I	943