CONTENTS

Preface v

PART ONE

INTRODUCTION 1

1 A PIAGETIAN FRAMEWORK FOR OBSERVING CHILDREN'S NUMERICAL THINKING 3

2 LISTENING TO CHILDREN: THE INTERVIEW METHOD 26

PART TWO

EARLY NUMBER CONCEPTS– Thinking, Learning, Teaching 39

3 COUNTING IN THE EARLY YEARS: CAPACITIES AND CONSTRAINTS 41

4 COUNTING AS A PROBLEM-SOLVING TOOL 57

5 DERIVED THINKING STRATEGIES: BASIS FOR A NETWORK OF NUMBER RELATIONS 93

6 ARITHMETIC NOTATION AND ITS INTERPRETATION 119

7 CONSTRUCTION OF MATHEMATICAL EQUALITY 152

8 INTERPRETATION OF NUMBER STORY PROBLEMS 191

9 TEACHING DIRECTIONS, DECISIONS, AND DILEMMAS 221

PART THREE

PLACE VALUE FOUNDATIONS AND APPLICATIONS– Thinking, Learning, Teaching 239

10 COUNTING BY ONES, TENS, AND HUNDREDS: MIRROR OF PLACE VALUE READINESS 241

11 INTERPRETATION OF THE PLACE VALUE SYSTEM 277

12 UNDERSTANDING OF THE STANDARD ADDITION ALGORITHM 308

13 UNDERSTANDING OF THE STANDARD SUBTRACTION ALGORITHM 339

14 INVENTION OF NONSTANDARD ALGORITHMS 373

PART FOUR

CONCLUSION 391

15 THE THREE FACES OF CLASSROOM COMPUTATION: AN ANALYSIS 393

16 TEACHERS AS ACTIVE AGENTS OF CHANGE 414

REFERENCES 425

APPENDICES

A THE PROGRESSION PUZZLE 431

B INTERVIEW HINTS 433

C THE TRANSITION FROM COUNTING-ALL TO COUNTING-ON IN ADDITION 437

D SYSTEM FOR TEACHING DERIVED STRATEGIES 440

E CATEGORIES OF NUMBER STORY PROBLEMS 441

F STARTING POINTS FOR INTERVIEWS ON OTHER AREAS OF NUMERICAL THINKING 442

G SOME RESOURCES FOR INFORMED CHANGE IN THE CLASSROOM 446

INDEX 447