## **Contents**

Preface			хi
Chapter	1	Points and Lines Connected with a Triangle	1
:	1.1	The extended Law of Sines	1
1	1.2	Ceva's theorem	4
1	1.3	Points of interest	7
1	1.4	The incircle and excircles	11
1	1.5	The Steiner-Lehmus theorem	14
1	1.6	The orthic triangle	16
:	1.7	The medial triangle and Euler line	18
1	1.8	The nine-point circle	20
1	1.9	Pedal triangles	22
Chapter	2	Some Properties of Circles	27
2	2.1	The power of a point with respect to a circle	27
2	2.2	The radical axis of two circles	31
2	2.3	Coaxal circles	35
2	2.4	More on the altitudes and orthocenter of a triangle	36
2	2.5	Simson lines	40
2	2.6	Ptolemy's theorem and its extension	42
2	2.7	More on Simson lines	43
2	2.8	The Butterfly	45
2	2.9	Morley's theorem	47
Chapter	3	Collinearity and Concurrence	51
3	3.1	Quadrangles; Varignon's theorem	51
3	3.2	Cyclic quadrangles; Brahmagupta's formula	56
3	3.3	Napoleon triangles	61

xiv CONTENTS

3.4	Menelaus's theorem	66
3.5	Pappus's theorem	67
3.6	Perspective triangles; Desargues's theorem	70
3.7	Hexagons	73
3.8	Pascal's theorem	74
3.9	Brianchon's theorem	77
Chapter 4	Transformations	80
4.1	Translation	81
4.2	Rotation	82
4.3	Half-turn	85
4.4	Reflection	86
4.5	Fagnano's problem	88
4.6	The three jug problem	89
4.7	Dilatation	94
4.8	Spiral similarity	95
4.9	A genealogy of transformations	100
Chapter 5	An Introduction to Inversive Geometry	103
5.1	Separation	103
5.2	Cross ratio	107
5.3	Inversion	108
5.4	The inversive plane	112
5.5	Orthogonality	114
5.6	Feuerbach's theorem	117
5.7	Coaxal circles	120
5.8	Inversive distance	123
5.9	Hyperbolic functions	126
Chapter 6	An Introduction to Projective Geometry	132
6.1	Reciprocation	132
6.2	The polar circle of a triangle	136
6.3	Conics	138
6.4	Focus and directrix	141
6.5	The projective plane	144
6.6	Central conics	146
6.7	Stereographic and gnomonic projection	150
Hints and An	swers to Exercises	154
References		181
Glossary		183
Index		189