## contents

	Preface	v
	PART 1 MATHEMATICS: CREATIVITY AND FORMALITY	
	Introduction to Part 1	1
l.	A small mathematical system	3
	An Abstract Mathematical System	3
	<ol> <li>An Underlying Language</li> <li>Vocabulary</li> <li>Assumptions</li> <li>A Deductive Logic and Theorems</li> </ol>	3 4 5 6
	Review of Set Language	6
	<ul><li>5. Basic Set Language</li><li>6. Basic Relations for Sets</li></ul>	6 8
	A Small Geometry	10
	<ol> <li>The Abstract System</li> <li>Interpretations and Models</li> <li>Theorems of the Small Geometry</li> </ol>	10 10 13
	Logic and proof	15
	Forming Statements in Logic	15
	<ol> <li>Sentences and Statements</li> <li>Connectives: And, Or, Not</li> <li>Connectives: Conditional</li> </ol>	15 16 19

## viii □ contents

	<ul><li>4. Special Statement Forms</li><li>5. Negation of Compound Statements</li></ul>	$\frac{20}{25}$
	Inference and Proof	28
	6. Basic Inference Schemes	28
	7. Definition of a Mathematical Proof	32
	Mention and Use	34
	8. Quotes	34
3.	Geometry of points, lines, and planes	36
	Basic Vocabulary of Euclidean Geometry	37
	1. Undefined Words	37
	First List of Postulates	37
	<ol> <li>Points and Lines Postulates</li> <li>Theorems Based on the First List of Postulates</li> </ol>	38 38
	Second List of Postulates	40
	4. Points and Planes 5. Theorems Based on the First and Second List of	41
	Postulates	41
	Third List of Postulates	42
	6. Lines and Planes	42 44
	7. Theorems Based on the Nine Postulates	48
	Models	46 48
	8. Interpretations and Models for the Nine Postulates	40
	PART 2 MEASUREMENT OF SETS OF POINTS	
	Introduction to Part 2	5
4.	Real numbers and line coordinates	53
	The Real Numbers	54
	1. Some Familiar Sets of Numbers	54
	2. Properties of the Real Numbers	54
	3. Properties of Equality, Subtraction, and Division	56 57
	<ol> <li>Properties of Order for Real Numbers</li> <li>Betweenness for Numbers, Density, and</li> </ol>	0.
	Completeness	60
	A Coordinate System on a Line	6
	6. A Physical Ruler	6
	7. Distance and Its Measure	6
	8. A Coordinate System on a Line	6
	9. Two Coordinate Systems on a Line	7. 7.
	<ul><li>10. Betweenness for Points and Segments</li><li>11. Midpoints of Segments</li></ul>	7
5.	Measurement	7
Э.	-	7
	Introductory Remarks	/

	1. Reliability of Measurements	78
	Conventions in Measurement	79
	<ol> <li>Precision and Greatest Possible Error</li> <li>Significant Digits, Relative Error, and Accuracy</li> </ol>	79 81
	Calculations with Measurements	84
6.	Angles and angle measurement	85
	Rays and Angles	85
	<ol> <li>Rays</li> <li>Rays and a Line-coordinate System</li> <li>Angles</li> </ol>	85 86 88
	Convex Sets and Angle Measurement	89
	<ol> <li>Convex Sets</li> <li>Separation</li> <li>Angle Measurement</li> <li>A Ray-coordinate System</li> <li>Betweenness for Rays</li> <li>Midray</li> <li>The Interior of an Angle</li> </ol>	89 90 91 93 96 96 98
	Kinds of Angles	100
	<ol> <li>Pairs of Angles</li> <li>Right Angles and Perpendicularity</li> <li>Complementary, Supplementary, and Vertical Angles</li> </ol>	100 102 103
7.	Area	106
	Triangles, Quadrilaterals, and Polygons	106
	<ol> <li>Basic Definitions for Triangles</li> <li>Quadrilaterals</li> <li>Polygons</li> </ol>	106 108 109
	Polygonal Regions and Their Areas	110
	<ul><li>4. A Polygonal Region</li><li>5. Measure of a Polygonal Region</li></ul>	110 112
	PART 3 CONGRUENCE, PARALLELISM, AND SIMILARITY OF SETS OF POINTS	115
	Introduction to Part 3	115
8.	Congruence	117
	Same Size and Shape	117
	1. An Experiment with Models	118
	Segments and Angles	118
	<ul><li>2. Congruence of Segments</li><li>3. Congruence of Angles</li></ul>	119 120

9.

10.

Tria	ngles	122
4.	Congruence between Triangles	123
5.	Postulates for Congruent Triangles	125
6.	Theorems That Follow from the Congruence	126
~	Postulates Medians, Altitudes, and Angle Bisectors of an	120
7.	Isosceles Triangle	129
8.	Exterior Angle of a Triangle	130
Para	allelism	134
	allel Lines	135
1.	Basic Definitions for Parallel Lines	135
	orems before a Parallel Postulate	138
2.	Proving Lines Parallel	138
	neuclidean Geometries	140
		141
3. 4.	Hyperbolic Geometry Elliptic Geometry	142
	corems after the Parallel Postulate	142
	Consequences of the Parallel Postulate	143
5. 6.	Impact of the Parallel Postulate on Triangles	144
	Quadrilateral Family	146
7.	Quadrilaterals	146
8.	Trapezoids	146
9.	Parallelograms	147
10.	When Is a Quadrilateral a Parallelogram?	148
11.		150 151
12.	Č	151 151
13.	Squares	
Tri	angle Inequalities	153
14.	Triangle-inequality Theorems	<i>15</i> 3
Sin	nilarity	155
Pro	pportion and Proportionality	155
1.		156
2.	*	157
Co	ncept of a Similarity	158
3.		159
4.		160
Th	e Pythagorean Theorem	164
<b>5</b> .		164
6.		166
Py	thagorean Triples	168
7.		168 170
8.	Special Right Triangles	170

11.	Area: polygonal regions and circular regions	174
	Areas of Common Regions	174
	1. Triangular Regions	175
	2. Parallelogram Regions	176
	3. Trapezoidal Regions	176
	Regular Polygonal Regions	179
	4. Regular Polygons	179
	5. Area of a Regular Polygonal Region	179
	Circular Regions	181
	6. Circles and Circular Regions	182
	7. Circumference of a Circle	182
	8. Area of a Circular Region	184
12.	Space figures: area and volume	187
	Lines and Planes in Space	187
	1. Perpendicularity of a Line and a Plane	187
	2. Parallelism of Lines and Planes	188
	Space Figures	190
	3. Polyhedrons	190
	4. Prisms	191
	5. Pyramids	194
	6. Cylinders	196 198
	7. Cones	
	Volumes of Space Figures	200
	8. Polyhedral Solids	200
	9. Volume of a Polyhedral Solid	201 202
	10. Volume of a Right Prism 11. Volume of a Pyramid	202
	12. Volume of a Right Circular Cylinder	205
	13. Volume of a Right Circular Cone	206
	PART 4 GEOMETRY OF TRANSFORMATIONS	209
	Introduction to Part 4	210
13.	Transformations of the plane: rigid motions	211
	Abstract versus "Physical" Geometry	211
	1. Physical Applications of Geometry	212
	What Is a Motion?	212
	2. Motion by Proxy: Correspondences	212
	3. Mappings: Onto and One-to-One	213
	4. Transformations	214
	Working with Transformations	214
	5. Combining Transformations	216
	6. The Notion of a Group of Transformations	218
	7. Subgroups of Groups of Transformations	218

14.

		990
	Translations	220
	8. Invariants under a Translation	$\begin{array}{c} 221 \\ 222 \end{array}$
	9. Combining Translations 10. The Group of Translations	223
	10. The Group of Translations Rotations of the Plane	225
	- · · · · · · · · · · · · · · · · · · ·	225
	11. When Is a Transformation a Rotation? 12. Invariants under a Rotation	226
	13. Combining Rotations and Translations	228
	Is the System of Rotations a Group?	229
	Reflections of the Plane	232
	14. Invariants under Reflections	232
	15. Sums of Reflections, Translations, and Rotations	233
	16. The Subgroup of Rigid Motions	239
14.	Similarity and other affine transformations	240
	Two-way Stretches	241
	1. Two-way Stretches and the Girdle Analogy	241
	2. Two-way Stretches of the Plane	241
	3. What Is a Two-way Stretch?	242
	4. Invariants under Two-way Stretches	244
	The Group of Euclidean Transformations	245
	5. Sums of Two-way Stretches	245 247
	6. Similarities	
	Nonuniform Two-way Stretches	248
	7. Stretches with Different Coefficients	249 250
	8. Invariants under Two-way Stretches	
	Affine Transformations	252 252
	9. Sum of a Rigid Motion and a Similarity	252
15.	Continuous transformations	254
	Relaxation of Previous Restrictions	254
	1. Examples of New Transformations	255
	2 Definition of Continuous Transformation	255
	3. The Group of Continuous Transformations	255
	Topology: The Study of Invariants under Continuous	
	Transformations	256
	4. Topological Equivalence	257
	5. Invariants under Topological Transformations	258
	6. Proof of the Jordan Curve Theorem for a Polygon	259 260
	7. Applications of the Jordan Curve Theorem 8. Fixed Points under Topological Transformations	261
	8. Fixed Points under Topological Transformations Answers to Odd-numbered Problems	264
		299
	Index	