CONTENTS

1. Introduction 1

1.1. Purposes of the Book 4
1.2. Organization of the Book 6

Part I – The Inferential and Decision Framework 9

2. Some Alternative Sampling Theory and Bayesian Linear Statistical Models, Estimators, and Tests 11

2.1. Decision Theory and Estimation 11
2.1.1. The framework 11
2.1.2. Choosing a decision rule 12
2.1.3. The squared error loss measure 15
2.1.4. The risk matrix measure 16
2.2. The Basic Linear Statistical Model and Some Traditional Estimators 17
2.2.1. The model 17
2.2.2. A best linear unbiased estimator 18
2.2.3. The prediction function problem 21
2.2.4. The reparameterized statistical model 22
2.2.5. Generalized least squares model and estimator 24
2.2.6. A multivariate specification 25
2.3. The General Linear Hypothesis Statistical Model and Estimator 26
2.3.1. The statistical model 26
2.3.2. The estimator 27
2.3.3. The risk matrix criterion 29
2.3.4. A weighted mean squared error risk 30
2.3.5. The conditional mean forecasting risk 33
2.4. Implications for Model Misspecification 34
2.5. The Stochastic Linear Hypothesis Statistical Model 37
2.5.1. The model 37
2.5.2. The estimator 37
2.5.3. Estimator comparison – Covariance and risk matrix 38
2.5.4. Reformulated statistical model 39

2.5.5. A comparison of risk functions for the conventional and stochastic restricted estimators 41
2.5.6. A comparison of weighted risk functions for conventional and stochastic restricted estimators 42
2.6. Bayesian Estimation for the Linear Model 43
2.6.1. Non-informative prior 46
2.6.2. An alternative prior distribution 47
2.7. Hypothesis Testing 49
2.7.1. General linear hypothesis testing 50
2.7.2. Stochastic linear hypothesis 55
2.7.3. Bayesian hypothesis testing 58
2.8. Concluding Remarks 59

Part II – Preliminary Test Estimators 61

3. The Preliminary Test Estimator for the Orthonormal Statistical Model 63

3.1. Introduction 63
3.2. The Statistical Models, Estimators, and Tests 63
3.2.1. Orthonormal model and estimators 63
3.2.2. Risk function comparisons 66
3.2.3. Test mechanism 67
3.2.4. The pre-test estimator 68
3.3. Analysis of the Risk Function for the Pre-test Estimator 69
3.3.1. The risk function 69
3.3.2. Comparison of the risk functions 72
3.3.3. Optimal critical values for testing 75
3.4. Mean and Covariance 79
3.5. Concluding Remarks 81

4. The Preliminary Test Estimator (General Case) 83

4.1. Introduction 83
4.2. Models, Estimators and Tests 85
4.3. The Risk Function of the Preliminary Test Estimator 87
4.3.1. A restatement of the test statistic 89
4.3.2. A reformulation of the risk function 89
4.3.3. Characteristics of the risk function 92
4.3.4. An alternative development of the pre-test loss function 93
4.4. Comparison of the Risk Functions 95
4.4.1. Conventional and preliminary test estimators 95
4.4.2. Preliminary test and restricted estimators 98
4.5. Optimal Choice of α 100
4.6. The Bias and Covariance Matrix 101
4.7. Conditional Mean Forecast Problem 102
4.8. Summary Remarks 103

5. The Performance of the Pre-test Estimator under a Risk Matrix Criterion 105

5.1. Introduction 105
5.2. The Statistical Models and Estimators 106
5.3. The Risk Matrix of the Preliminary Test Estimator 106
5.3.1. A reformulation of the test statistic 107
5.3.2. A restatement of the pre-test risk matrix 108
5.3.3. The risk matrix in terms of $\hat{\theta}$ 111
5.3.4. Comparison of the risk matrices 112
5.4. The Mean and Covariance Matrix of the Pre-test Estimator 115
5.5. Some Alternative Risk Matrix Comparisons 117
5.6. The Forecast Risk Matrix 118
5.7. A Concluding Remark 119

6. The Stochastic Hypothesis Preliminary Test Estimator 121

6.1. Introduction 121
6.2. Statistical Models, Estimators and Tests 121
6.3. A Stochastic Hypothesis Preliminary Test Estimator 124
6.4. A Comparison of the Risk Matrices of the Conventional and Stochastic Hypothesis Preliminary Test Estimators 125
6.4.1. The risk matrix 126
6.4.2. Conditions for the Δ matrix to be positive semi-definite 128
6.5. A Comparison of the Risk Functions of the Conventional and Preliminary Test Estimators 129
6.6. Mean and Covariance of the Preliminary Test Estimator 131
6.7. Some Stochastic Hypotheses Test Generalizations and an Application 132
6.7.1. Prior information via introspection 133
6.7.2. Prior information from a previous sample 134
6.7.3. Zellner's seemingly unrelated estimates and an introspective prior 136
6.7.4. An application 139

7. Autocorrelation Pre-test Estimators 143

7.1. The Statistical Model 144
7.2. Alternative Estimators under Autocorrelation 145
7.2.1. Generalized least squares 146
7.2.2. Cochrane–Orcutt two-stage procedure 146
7.2.3. The Durbin estimator 147
7.2.4. The Prais–Winsten estimator 147
7.3. Tests for Serial Correlation 148
7.3.1. Durbin and Watson 148
7.3.2. The Berenblutt and Webb g-statistic 149
7.3.3. Other test statistics 151
7.4. A Two-stage Estimator 151

7.5. The Experimental Design 152
7.6. Empirical Results 153
7.6.1. The Durbin–Watson pre-test risks 154
7.6.2. The Berenblutt and Webb pre-test risk 156
7.7. Conclusions 160
7.8. Tables of Empirical Risk Function Values 161

Part III – The Stein-rule Family of Estimators 165

8. Introduction to the Stein-rule Estimators 167

8.1. Introduction 167
8.2. The Statistical Model and Maximum Likelihood Estimator 167
8.3. A Stein-rule Estimator 170
8.3.1. The James and Stein estimator 170
8.3.2. Mean and covariance matrix 172
8.3.3. An interpretation of the James and Stein estimator 173
8.4. Some Variations of the Stein-rule 176
8.4.1. A reformulation 176
8.4.2. The case of unknown σ^2 176
8.4.3. The Baranchik result 181
8.5. The James and Stein Positive-rule Estimator 182
8.5.1. Generalized positive-rule estimators 185
8.6. The Strawderman Result 187
8.7. Sclove's Modified Positive-rule Estimator 189
8.7.1. Risk comparisons 192
8.8. Combining Normal Means and Regression Equations 195
8.8.1. The Zellner model 195
8.8.2. Combining three or more independent normal mean estimation problems with unknown variances 197
8.8.3. Combining regression equations with unknown variances 198
8.9. Appendix to Chapter 8 200
8.9.1. The covariance matrix of the James and Stein estimator, σ^2 known 200
8.9.2. The covariance matrix of the James and Stein estimator, σ^2 unknown 201
8.9.3. The covariance matrix for the positive-rule estimator, σ^2 unknown 203
8.9.4. The covariance matrix of the Sclove modified Stein-rule estimator 204

9. Some Monte Carlo Sampling Results – Identity Covariance 207

9.1. Empirical Risk Functions for the Sclove Modified Stein-rule Estimator 207
9.1.1. The sampling experiment 208
9.1.2. The sampling results 209

9.1.3. Concluding remarks 213
9.2. Family of Positive-rule Estimators, σ^2 Known 214
9.3. A Monte Carlo Experiment for a Family of Stein-rule Estimators, σ^2 Unknown 221
9.4. Sampling Distribution of the Stein-rules 223

10. Stein-rule Estimators under Weighted Squared Error Loss 229

10.1. The Statistical Model and Loss Function 229
10.2. General Minimax and Positive-rule Estimators 230
10.2.1. General minimax estimator 231
10.2.2. General positive-rule estimators 238
10.3. Some Alternatives from the Family of Stein-rule Estimators 240
10.3.1. A James and Stein type minimax estimator 240
10.3.2. An alternative estimator 248
10.3.3. Berger's minimax estimator 250
10.4. Admissible Minimax Estimators 253
10.4.1. The Bock–Strawderman estimator 253
10.4.2. The Berger–Strawderman estimator 254
10.5. Combining Parameter Vectors 255

11. Some Monte Carlo Sampling Results – Non-identity Covariance 259

11.1. The Experimental Design 259
11.2. James and Stein Form of the Minimax Estimator 261
11.3. Berger–Strawderman and Stein-rule Sampling Results 265
11.4. Diagonal Covariance Matrix with Unknown Variances 268
11.5. Weighted Risk Function with Unknown Covariance Matrix 270
11.6. Stein-rules with All Elements of the Covariance Matrix Unknown 271
11.7. Concluding Remarks 274

12. The Stochastic Regressor Statistical Model 275

12.1. Introduction 275
12.2. The Statistical Model 275
12.3. The Measure of Goodness 277
12.4. The Baranchik–Stein Prediction Function 277
12.5. An Alternative Prediction Function 281
12.6. Some Final Remarks 287

13. Some Applications of the Estimators to Economic Data 293

13.1. Introduction 293
13.2. Price–cost Margins and Industry Structure 294
13.3. The Phillips Curve 300
13.4. Evaluation and Concluding Remarks 307

14. Some Final Remarks 309

Appendix A. Some Distribution and Matrix Theorems 313

A.1. Normal Distributions 313
A.2. Chi-square Distributions 313
A.3. Matrix Algebra Relevant to Normal Theory 315

Appendix B. Theorems and Lemmas Used in the Risk Evaluations 319

B.1. Some Supporting Lemmas 319
B.2. Some Basic Theorems 321
B.3. Properties of Functions of the Non-central F Distribution 327

References 331

Subject Index 339