

Contents

Preface	v
List of Figures	xi
1 Pricing by Arbitrage	1
1.1 Introduction: Pricing and Hedging	1
1.2 Single-Period Option Pricing Models	9
1.3 A General Single-Period Model	12
1.4 A Single-Period Binomial Model	13
1.5 Multi-Period Binomial Models	17
1.6 Bounds on Option Prices	21
2 Martingale Measures	23
2.1 A General Discrete-Time Market Model	23
2.2 Trading Strategies and Arbitrage Opportunities	25
2.3 Martingales and Risk-Neutral Pricing	30
2.4 Arbitrage Pricing with Martingale Measures	32
2.5 Example: Martingale Formulation of the Binomial Market Model	35
2.6 From CRR to Black–Scholes	38
3 The Fundamental Theorem of Asset Pricing	45
3.1 The Separating Hyperplane Theorem in \mathbb{R}^n	45
3.2 Construction of Martingale Measures	47

3.3	A Local Form of the ‘No Arbitrage’ Condition	49
3.4	Two Simple Examples	56
3.5	Equivalent Martingale Measures for Discrete Market Models	59
4	Complete Markets and Martingale Representation	63
4.1	Uniqueness of the EMM	63
4.2	Completeness and Martingale Representation	65
4.3	Martingale Representation in the CRR-Model	66
4.4	The Splitting Index and Completeness	70
4.5	Characterisation of Attainable Claims	73
5	Stopping Times and American Options	75
5.1	Hedging American Claims	75
5.2	Stopping Times and Stopped Processes	77
5.3	Uniformly Integrable Martingales	80
5.4	Optimal Stopping; The Snell Envelope	86
5.5	Pricing and Hedging American Options	93
5.6	Consumption–Investment Strategies	96
6	A Review of Continuous-Time Stochastic Calculus	99
6.1	Continuous-Time Processes	99
6.2	Martingales	103
6.3	Stochastic Integrals	109
6.4	The Itô Calculus	117
6.5	Stochastic Differential Equations	126
6.6	The Markov Property of Solutions of SDEs	129
7	European Options in Continuous Time	135
7.1	Dynamics	135
7.2	Girsanov’s Theorem	136
7.3	Martingale Representation	142
7.4	Self-Financing Strategies	151
7.5	An Equivalent Martingale Measure	154
7.6	The Black–Scholes Formula	163
7.7	A Multi-Dimensional Situation	167
7.8	Barrier Options	172
8	The American Option	187
8.1	Extended Trading Strategies	187
8.2	Analysis of American Put Options	190
8.3	The Perpetual Put Option	196
8.4	Early Exercise Premium	199
8.5	Relation to Free Boundary Problems	202
8.6	An Approximate Solution	208

9	Bonds and Term Structure	211
9.1	Market Dynamics	211
9.2	Future Price and Futures Contracts	215
9.3	Changing Numéraire	219
9.4	A General Option Pricing Formula	222
9.5	Term Structure Models	227
9.6	Diffusion Models for the Short-Term Rate Process	229
9.7	The Heath–Jarrow–Morton Model	242
9.8	A Markov Chain Model	247
10	Consumption-Investment Strategies	251
10.1	Utility Functions	251
10.2	Admissible Strategies	253
10.3	Utility Maximization from Consumption	258
10.4	Maximization of Terminal Utility	263
10.5	Utility Maximization for Both Consumption and Terminal Wealth	266
	References	271
	Index	289