

# Contents

Preface	<i>vii</i>
Bibliography	<i>xv</i>

## PART I THE PREHISTORICAL PERIOD *1*

### CHAPTER I **Counting**

1	Number Sense	<i>3</i>
2	Tallying	<i>5</i>
3	Vocal Sounds for "Concrete" Numbers	<i>7</i>
4	Abstract Number Words	<i>7</i>
5	Extension of Abstract Number Words	<i>7</i>
6	Finger Counting	<i>8</i>
7	Numerals	<i>10</i>
	7:1 Differences between Numbers and Numerals	<i>10</i>
	7:2 Creation of Numerals	<i>11</i>
8	Numeral Systems	<i>11</i>
	8:1 Development of a Simple Grouping Numeral System	<i>11</i>
	8:2 Development of a Positional Numeral System	<i>12</i>
	Exercises	<i>13</i>

CHAPTER II **Sets**

- 1 The Set Concept 16
  - 1:1 "Well-defined" Concept 16
  - 1:2 Set Membership and Notation 17
- 2 Methods for Defining Particular Sets 17
  - 2:1 Definition by Listing 17
  - 2:2 Definition by Description 17
- 3 Special Sets 18
  - 3:1 Empty Set 18
  - 3:2 Universal Set 18
- 4 Relations between Sets 19
  - 4:1 Subsets 19
  - 4:2 Equal Sets 19
  - 4:3 Equivalent Sets 19
- 5 Venn Diagrams 20
- 6 Operations on Sets 21
  - 6:1 Complementation 21
  - 6:2 Union 22
  - 6:3 Intersection 22
- 7 Combinations of Operations 23
- 8 Extension to Three Sets 24
- 9 Applications to Survey Problems 25
- 10 Number 26
  - 10:1 Cardinal Numbers 26
  - 10:2 Counting 27
  - 10:3 Ordinal Numbers 27
  - Exercises 28

PART II **THE ANCIENT ORIENTAL PERIOD** 33CHAPTER III **Numeral Systems**

- 1 Simple Grouping Numeral Systems 37
  - 1:1 Egyptian Hieroglyphic Numerals 37
  - 1:2 Babylonian Cuneiform Numerals 39
  - 1:3 Attic Greek Numerals 40
  - 1:4 Roman Numerals 41
  - 1:5 Aztec Numerals (Optional) 42
- 2 Ciphred Numeral Systems 43
  - 2:1 General Discussion 43
  - 2:2 Ionic Greek Numerals 44
- 3 Multiplicative Grouping Numeral Systems 45
  - 3:1 General Discussion 45
  - 3:2 Traditional Chinese Numerals 46
  - 3:3 Tamil Numerals (Optional) 47

4	Positional Numeral Systems	47
4:1	Babylonian Numerals	48
4:2	Scientific Chinese Numerals	50
4:3	Maya Numerals	50
4:4	Hindu-Arabic Numerals	52
5	Development of a Numeral System	53
	Exercises	54
CHAPTER IV	<b>Arbitrary Bases</b>	58
1	Exponential Notation	59
1:1	Decimal Notation	59
1:2	Bases Other Than Ten	60
2	Conversion to and from Base 5	60
2:1	Conversion to Base 10	60
2:2	Conversion from Base 10	61
3	Conversion of Arbitrary Bases	62
3:1	To Base 10	62
3:2	From Base 10	62
4	Computation in Arbitrary Bases	63
4:1	Addition and Subtraction in Base 5	63
4:2	Multiplication in Base 5	64
4:3	Division in Base 5 (Optional)	65
4:4	Computation in Arbitrary Bases	66
5	Basimal Fractions (Optional)	67
5:1	Quinary Fractions	67
5:2	Fractions, Arbitrary Bases	68
	Exercises	70
CHAPTER V	<b>Early Computations</b>	73
	<b>Prealgebra</b>	73
1	Abacus	73
2	Egyptian Addition and Subtraction	75
3	Multiplication by Duplation	76
4	Fractions, Division	77
4:1	Egyptian Unit Fractions	77
4:2	Babylonian Sexagesimal Fractions	78
5	Miscellaneous Problems	80
5:1	Rule of False Position	80
5:2	Special Quadratic and Cubic Equations (Optional)	81
5:3	Compound Interest Problems (Optional)	81
5:4	Progressions (Optional)	82
	<b>Pregeometry</b>	83
1	Introduction to Geometric Figures	83

2	Formulas for Areas	85
2:1	Squares, Rectangles, Right Triangles	85
2:2	Isosceles Triangles	86
2:3	Trapezoids	87
2:4	Circles	88
3	Formulas for Volumes	88
3:1	Cubes and Boxes	88
3:2	Cylinders	89
3:3	Truncated Pyramid	89
4	Pythagorean Numbers and the Right Triangle	90
	Exercises	92

### PART III **THE GREEK PERIOD** 95

#### CHAPTER VI **Logic** 106

1	Short History of Logic	106
2	Aristotelian Logic	107
2:1	Syllogisms	107
2:2	Difficulties	108
3	Symbolic Logic	108
3:1	Tf Statements	108
3:2	Connectives	109
3:3	Truth Tables and Set Diagrams	109
3:4	Word Translations	112
4	Tautologies	113
4:1	Definition and Examples	113
4:2	Statement, Converse, Inverse, Contrapositive	113
4:3	DeMorgan's Laws	114
4:4	Other Tautologies (Optional)	115
5	Methods of Proof	115
5:1	Proof Patterns	115
5:2	Validity	117
5:3	Combinations of Patterns	118
	Exercises	119

#### CHAPTER VII **Mathematical Systems** 126

1	General Discussion	126
1:1	Undefined Concepts and Definitions	126
1:2	Assumptions and Theorems	127
1:3	Formal Systems	128
2	Finite Algebras	130
2:1	Clock Arithmetic	130
2:2	Boolean Algebra (Optional)	133
3	Finite Geometries (Optional)	138
	Exercises	141

<b>CHAPTER VIII Elementary Euclidean Geometry</b>	<b>147</b>
1 Euclid's Ten Assumptions	147
2 Constructions	149
3 Modern Modifications	151
4 Modern Introduction to Euclidean Geometry	152
5 Theory of Parallels and Indirect Proof	164
5:1 Indirect Proof	164
5:2 Theory of Parallels	165
6 Areas (Optional)	166
7 Theorem of Pythagoras	169
8 Similar Triangles	170
9 Eratosthenes and Earth Measurement (Optional)	173
10 Archimedes (Optional)	174
Exercises	175
<b>CHAPTER IX Greek Arithmetic (Optional)</b>	<b>181</b>
1 Polygonal Numbers	181
2 Euclidean Algorithm	182
2:1 Greatest Common Divisor	182
2:2 Euclidean Algorithm	183
3 Perfect and Friendly Numbers	184
3:1 Perfect Numbers	184
3:2 Friendly Numbers	184
4 Prime Numbers	185
4:1 Euclid's Theorem	185
4:2 Sieve of Eratosthenes	186
4:3 Twin Primes and Unproved Conjectures	187
5 Geometric Algebra	188
6 Diophantus	189
Exercises	191
<b>PART IV THE HINDU-ARABIC-EUROPEAN PERIOD</b>	<b>193</b>
<b>CHAPTER X The Development of Arithmetic</b>	<b>211</b>
1 Computations by Abacus	212
2 Computations by Algorithms	213
2:1 Addition (The Union of Disjoint Sets)	213
2:2 Subtraction (The Inverse of Addition)	217
2:3 Multiplication (The Cartesian Product of Two Sets)	219
2:4 Division (The Inverse of Multiplication)	225
3 Problem Solving by Inversion	228
Exercises	230

CHAPTER XI	<b>Algebra</b>	233
1	Development of the Number System	234
1:1	Positive Integers and Early Fractions	234
1:2	Irrational Numbers	235
1:3	Common Fractions	235
1:4	Decimal Fractions	241
1:5	Negative Numbers and the Rationals	244
1:6	Complex Numbers	247
2	Development of Name and Symbols	249
2:1	Meaning of Algebra	249
2:2	Name "Algebra"	249
2:3	Development of the Symbols	250
3	Elementary Algebra as an Axiomatic System	252
4	Solution of Linear Equations	257
5	Graphs of Linear Equations	259
6	Simultaneous Linear Equations	261
	Exercises	263
CHAPTER XII	<b>Probability, Calculus, Geometries (Optional)</b>	266
1	Probability	266
1:1	Meaning of Probability	266
1:2	Combinations of Events	268
1:3	Repeated Trials and Pascal's Triangle	271
1:4	Statistical Inference	272
	Exercises	273
2	Analytic Geometry and Calculus	275
2:1	Analytic Geometry	275
2:2	Integral Calculus	277
2:3	Differential Calculus (The Tangent Problem)	281
2:4	Fundamental Theorem of Calculus	285
3	Geometries	287
3:1	Non-Euclidean Geometries	287
3:2	Topology	289
	Historical Time Chart	297
	Answers to Selected Exercises	303
	Index	321