Contents

LIST OF SYMBOLS viii
INTRODUCTION ix

1 The Slope of the Tangent Line 1

the graph of the train's position 3
function machines 4
slope of a line 5
slope of a secant line 7
slope of a tangent line 9
tangent slope for $y = x^2$ 9
EXERCISES 11

2 Calculating Derivatives 12

definition of derivative 13
derivative of a constant function 14
derivative of a straight-line function 16
derivative of $y = cx^2$ 17
derivative of a sum 20
derivative of $y = x^3$ 21
derivative of $y = x^n$ 24
EXERCISES 26

3 Drawing Curves with Derivatives 28

horizontal tangents 30
the derivative of the derivative 32
the professor's bug and the meaning of the second derivative 34
concave-up and concave-down curves 35
the spilled water 36
local maximum points 37
points of inflection 38
EXERCISES 40

4 Derivatives of Complicated Functions 41

multiplied functions 44
the product rule 47
embedded functions 49
the chain rule 50
fractional exponents 51
implicit functions 51
the power rule 53
EXERCISES 56

5 Derivatives of Trigonometric Functions 57

the gremlin's horrible oscillating chicken-scaring machine 58
Trigonometeris' sine function 60
the derivative of the sine function 64
the derivative of the cosine function 65
derivatives of other trigonometric functions 66
EXERCISES 67

6 Optimum Values and Related Rates 69

differentiation and the get-rich-quick scheme 70
the optimum-size box 71
Carmorra Magazine and the optimum subscription price 72
the birthday party balloon 75
the National Park Beach lifeguard and the racing shadow 76
EXERCISES 78

7 The Integral: A Backward Derivative 80

Recordis' exhaustion and the story of Rutherford 80
differentiating backwards 81
the antiderivative or the integral 82

discovering the indefiniteness of an indefinite integral 83
using an initial condition to track down Rutherford 84
differentials 85
the integral sign 85
sum rule for integrals 87
multiplication rule for integrals 88
perfect integral rule 89
power rule for integrals 90
EXERCISES 92

8 Finding Areas with Integrals 95

Recordis' pools and the Magic Crystal Water rate increase 96
summation notation 97
the curve's area defined as a limit 99
the gremlin's terrible fire-and-water threat 99
the mysterious function $A(x)$ 100
the derivative of $A(x)$ 102
fundamental theorem of integral calculus 103
discovering the definiteness of definite integrals 103
EXERCISES 105

9 Natural Logarithms 108

the unfortunate accident with some beads 109
the power rule breakdown: $n = -1$ 110
the mysterious function $L(a)$ 112
some properties of $L(a)$ 113
substitution method for evaluating definite integrals 115
remembering logarithms 116
the derivative of the logarithm function 117
the fundamental number e 118
EXERCISES 122

10 Exponential Functions and Integration by Parts 124

the graph of the logarithm function 125
Mongol's stumble and the inverse function 125
the exponential function and the professor's amazing income 126
the indestructible function e^x 128
differentiation of exponential functions 129
the method of logarithmic implicit differentiation 129
the integral of the logarithm function 132
the method of integration by parts 133
EXERCISES 135

11 Integration by Trigonometric Substitution 137

the elliptical rose garden 137
the ellipse area integral 138
trying a trigonometric substitution 139
the area of the ellipse 141
the method of trigonometric substitution 142
derivatives of inverse trigonometric functions 143
EXERCISES 144

12 Integration by Partial Fractions 146

the red-and-yellow fireworks problem 148
the integral of the secant function 153
partial fractions with quadratic denominators 155
the method of partial fractions 158
EXERCISES 160

13 Finding Volumes with Integrals 163

the pancake method of approximating volume 164
the amazing resemblance between the continuous sum and the definite integral 166
the volume of the paraboloid 167
finding volumes with cylindrical shells 170
EXERCISES 176

14 Arc Lengths, Surface Areas, and the Center of Mass 177

the straight-line approximation for a curve 178
the integral for arc lengths 180
the frustum method of finding surface areas 185
the center of mass of the concert hall stage 187
EXERCISES 190

15 Introduction to Differential Equations 192

the oscillating ride and the ordinary differential equation 193
linear differential equations 195
the force of friction and the damped sine wave 202
solution method for second-order linear homogeneous constant-coefficient differential equations 204
the driving force and the nonhomogeneous equation 205
resonance and the infinite amplitude ride 210
EXERCISES 212

16 Comprehensive Test of Calculus Problems 214

the return of the gremlin 214
the 45 problems 215
the solutions 218

17 Stanislavsky Guide to Calculus 239

Appendix: Answers to exercises 243
INDEX 271