Contents

Preface iii

Programming notes vii

1 Factors and prime numbers 1

Factors Primes Prime factors Perfect numbers Eratosthenes' sieve Goldbach's conjecture Fundamental theorem of arithmetic Prime number theorems and conjectures

2 Euclid's algorithm 18

Highest common factors Lowest common multiples 'Division algorithm' Game of Euclid

3 Fractions and Euclid's algorithm 27

Egyptian fractions Farey sequences

4 Fibonacci sequences and the golden ratio 36

Golden rectangle Extending Fibonacci sequences Relations and theorems involving Fibonacci sequences

5 Decimal expansions of fractions 49

Long division Recurring decimals Fermat's little theorem Periodicity of recurring decimals Patterns

6 Binary expansions of decimals 63

Why do computers sometimes get the simplest sums wrong?

7 Approximating decimals by fractions 67

Non-terminating, non-recurring decimal numbers Continued fractions and basic theorems Applications

8 Square roots and cube roots 90

Squaring a rectangle Cubing a cuboid Cube roots using square roots

9 Pythagorean triples 98

Generating triples General solution of $x^2 + y^2 = z^2$ Consideration of $x^4 + y^4 = z^2$ Representation as sums of two squares, sums of three squares and sums of four squares (Lagrange's theorem)

10 Solving f(x) = 0 109

Simple iteration Bisection method Method of false position Newton-Raphson method Convergence of iterative procedures

11 Dividing without division 135

A 'fun' algorithm, but with good mathematics behind it

12 Linear simultaneous equations 138

Jacobi method Gauss-Seidel method

13 Instant interest 150

Compound, nominal and effective rates of interest Exponential function and series e and its irrationality Natural logarithm (ln)

14 Logarithms 160

Calculation of logarithms to base 10 Proofs of first mean-value theorem and Taylor's theorem Logarithmic functions expressed as Taylor series

15 Taylor series 169

Representation of $\sin x$, $\cos x$, $\tan^{-1} x$, $\ln (1 + x)$, $\exp (x)$ as Taylor series Numerical approximations Graphs of series approximations

16 Numerical integration 186

Approximation by rectangles Trapezium rule Mid-point method Simpson's rule Degrees of accuracy of approximations

17 Random numbers 200

Pseudo-random numbers Simulation Proofs Use of Riemann zeta function

Answers 219

Index 247