Contents

Preface *page* ix

I Locally integrable structures **1**

I.1 Complex vector fields 1
I.2 The algebraic structure of $\mathfrak{X}(\Omega)$ 4
I.3 Formally integrable structures 5
I.4 Differential forms 7
I.5 The Frobenius theorem 11
I.6 Analytic structures 14
I.7 The characteristic set 15
I.8 Some special structures 15
I.9 Locally integrable structures 19
I.10 Local generators 21
I.11 Local generators in analytic structures 25
I.12 Integrability of complex and elliptic structures 26
I.13 Elliptic structures in the real plane 28
I.14 Compatible submanifolds 32
I.15 Locally integrable CR structures 36
I.16 A CR structure that is not locally integrable 38
I.17 The Levi form on a formally integrable structure 42
Appendix: Proof of the Newlander–Nirenberg theorem 47
Notes 51

II The Baouendi–Treves approximation formula **52**

II.1 The approximation theorem 52
II.2 Distribution solutions 63
II.3 Convergence in standard functional spaces 69
II.4 Applications 83
Notes 99

III Sussmann's orbits and unique continuation **101**
III.1 Sussmann's orbits 101
III.2 Propagation of support and global unique continuation 108
III.3 The strong uniqueness property for locally integrable solutions 120
III.4 Proof of Theorem III.3.15 126
III.5 Uniqueness for approximate solutions 132
III.6 Real-analytic structures in the plane 140
III.7 Further applications of Sussmann's orbits 146
Notes 147

IV Local solvability of vector fields **149**
IV.1 Planar vector fields 150
IV.2 Solvability in C^∞ 176
IV.3 Vector fields in several variables 184
IV.4 Necessary conditions for local solvability 199
Notes 214

V The FBI transform and some applications **218**
V.1 Certain submanifolds of hypoanalytic manifolds 218
V.2 Microlocal analyticity and the FBI transform 226
V.3 Microlocal smoothness 236
V.4 Microlocal hypoanalyticity and the FBI transform 239
V.5 Application of the FBI transform to the C^∞ wave front set of solutions of nonlinear PDEs 243
V.6 Applications to edge-of-the-wedge theory 254
V.7 Application to the F. and M. Riesz theorem 263
Notes 270

VI Some boundary properties of solutions **271**
VI.1 Existence of a boundary value 271
VI.2 Pointwise convergence to the boundary value 281
VI.3 One-sided local solvability in the plane 286
VI.4 The H^p property for vector fields 289
Notes 307

VII The differential complex associated with a formally integrable structure **308**
VII.1 The exterior derivative 308
VII.2 The local representation of the exterior derivative 309
VII.3 The Poincaré Lemma 311
VII.4 The differential complex associated with a formally integrable structure 311

VII.5 Localization 312
VII.6 Germ solvability 313
VII.7 $\mathcal{V}$-cohomology and local solvability 315
VII.8 The Approximate Poincaré Lemma 316
VII.9 One-sided solvability 319
VII.10 Localization near a point at the boundary 321
VII.11 One-sided approximation 322
VII.12 A Mayer–Vietoris argument 323
VII.13 Local solvability versus local integrability 328
Notes 330

VIII Local solvability in locally integrable structures 331
VIII.1 Local solvability in essentially real structures 332
VIII.2 Local solvability in the analytic category 332
VIII.3 Elliptic structures 333
VIII.4 The Box operator associated with D 337
VIII.5 The intersection number 340
VIII.6 The intersection number under certain geometrical assumptions 343
VIII.7 A necessary condition for one-sided solvability 346
VIII.8 The sufficiency of condition $(\star)_0$ 348
VIII.9 Proof of Proposition VIII.8.2 350
VIII.10 Solvability for corank one analytic structures 354
Notes 357

Epilogue 361
1 The similarity principle and applications 361
2 Mizohata structures 364
3 Hypoanalytic structures 370
4 The local model for a hypoanalytic manifold 371
5 The sheaf of hyperfunction solutions on a hypoanalytic manifold 372

Appendix A Hardy space lemmas 374
A.1 Multipliers in h^1 374
A.2 Commutators 376
A.3 Change of variables 378

Bibliography 381

Index 390