## **Contents**

Preface			
Acl	xi xvii		
Inti			
	1. 2.	Cohomology and quantum cohomology Differential equations and D-modules	xvii xx
	3.	Integrable systems	xxii
1	The many faces of cohomology		1
	1.1	Simplicial homology	2
	1.2	Simplicial cohomology	3
	1.3	Other versions of homology and cohomology	4
	1.4	How to think about homology and cohomology	6 7
	1.5 1.6	Notation The symplectic volume function	10
2	Quantum cohomology		12
	2.1	3-point Gromov–Witten invariants	12
	2.2	The quantum product	16
	2.3	Examples of the quantum cohomology algebra	19
	24	Homological geometry	29

3	Quantum differential equations		
	3.1 3.2 3.3	The quantum differential equations Examples of quantum differential equations Intermission	33 39 43
4	Linear differential equations in general		
	4.1 4.2 4.3 4.4 4.5 4.6	Ordinary differential equations Partial differential equations Differential equations with spectral parameter Flat connections from extensions of D-modules Appendix: connections in differential geometry Appendix: self-adjointness	46 53 62 67 71 89
5	The	100	
	5.1 5.2 5.3 5.4	The quantum D-module The cyclic structure and the <i>J</i> -function Other properties Appendix: explicit formula for the <i>J</i> -function	100 102 106 112
6	Abst	116	
	6.1 6.2 6.3 6.4 6.5 6.6 6.7 6.8	The Birkhoff factorization Quantization of an algebra Digression on D <sup>h</sup> -modules Abstract quantum cohomology Properties of abstract quantum cohomology Computations for Fano type examples Beyond Fano type examples Towards integrable systems	116 124 125 130 135 138 144
7	Integrable systems		
	7.1 7.2	The KdV equation The mKdV equation	155 160

			Contents
	7.3	Harmonic maps into Lie groups	164
	7.4	Harmonic maps into symmetric spaces	171
	7.5	Pluriharmonic maps (and quantum cohomology)	176
	7.6	Summary: zero curvature equations	178
8	Solving integrable systems		
	8.1	The Grassmannian model	183
	8.2	The fundamental construction	186
	8.3	Solving the KdV equation: the Guiding Principle	191
	8.4	Solving the KdV equation	197
	8.5	Solving the KdV equation: summary	202
	8.6	Solving the harmonic map equation	206
	8.7	D-module aspects	218
	8.8	Appendix: the Birkhoff and Iwasawa decompositions	219
9	Quantum cohomology as an integrable system		223
	9.1	Large quantum cohomology	224
	9.2	Frobenius manifolds	229
	9.3	Homogeneity	236
	9.4	Semisimple Frobenius manifolds	239
10	Integrable systems and quantum cohomology		
	10.1	Motivation: variations of Hodge structure (VHS)	244
	10.2	Mirror symmetry: an example	255
		h-version	265
		Loop group version	270
		Integrable systems of mirror symmetry type	276
	10.6	Further developments	287
Ref	erenc	es	293
			222
Ind	ex		303