CONTENTS

Preface, xv

PART ONE. TRUTH-FUNCTIONAL LOGIC

- 1 Preliminaries, 3
 - 1.0 Scope of Text, 3
 - 1.1 Symbolic Logic; the Logistic Method, 4
- 2 Truth-Functional Sentence Connectives (I), 7
 - 2.0 Conjunction, 7
 - 2.1 Disjunction or Alternation, 9
 - 2.2 Negation, 10
- 3 Exercises, 11
- 4 Language Schema P, 12
 - 4.0 Vocabulary, 12
 - 4.1 Formation Rules, 12
 - 4.2 Use and Mention; Quotation Marks and Corners, 15
- 5 Exercises, 17
- 6 Semantics of Language Schema P, 18
 - 6.0 Semantics, 18
 - 6.1 Truth Tables, 19
 - 6.2 Interpreting Language Schema P, 25
- 7 Exercises, 27
- 8 Logical Truth and Analyticity, 29
 - 8.0 Logical Truth and Logical Falsehood, 29
 - 8.1 Analytic and Synthetic Sentences, 30
- 9 Exercises, 33
- 10 Abbreviation and Equivalence, 34
 - 10.0 Some Abbreviative Conventions, 34
 - 10.1 Equivalence, 36
 - 10.2 Tolerably Ambiguous Abbreviations, 38
- 11 Exercises, 40
- 12 Functional Completeness, 42

viii Contents

12.0	Mutual Entailment, Equivalence, and Expressive Power, 42
12.1	Functional Completeness of the Tilde, Wedge, and Dot, 43
Trut	h-Functional Sentence Connectives (II), 46
13.0	Truth-Functional Sentence Connectives; a Second
	Look, 46
13.1	Redundancy of the Tilde, Wedge, and Dot, 48
13.2	Sheffer's Stroke, 49
Impl	ication and Equivalence, 52
14.0	The Conditional, 52
14.1	The Biconditional, 54
14.2	Implication; Consequence Relation, 56
14.3	Short-cut Test for Validity and Implication, 58
14.4	More Abbreviative Conventions, 60
14.5	Equivalence and Implication, 61
	cises, 62
Norn	nal Forms; Duality, 64
16.0	- · · · · · · · · · · · · · · · · · · ·
16.1	Reduction to Normal Form, 67
$16.2 \\ 16.3$	Simple Disjunctive Normal Form, 71
	• ,
	ises, 74
B00le	an Equations; Electrical Circuits, 76
18.0	Boolean Equations, 76
18.1	Design of Electrical Circuits, 81 ·
	ises, 86
Appli	cation of Formalized Languages to the Logical Analysis
20.0	ourar hanguages, 89
20.0 20.1	Proving Correctness of Arguments, 89
20.1	Proving Incorrectness of Arguments, 93
20.3	Rendering Logical Structure Explicit, 94 Bringing Logical Form to the Grant Structure of th
20.4	Bringing Logical Form to the Surface, 101 The Semantics of Atomic Sentences, 104
	ises, 107
	ional Incompleteness, 109
22.0	Mathematical Induction, 109
22.1	Strong Mathematical Induction, 112
	S THE THOUGH THOUGHOIL, 11Z

- 22.2 Functional Incompleteness of the Dot and Wedge, 115
- 23 Exercises, 117
- 24 Alternative Notations, 118
 - 24.0 Trivial Alternatives, 118
 - 24.1 Nontrivial Alternative: Polish Notation, 119
- 25 Exercises, 122

PART TWO. AXIOMATIZATION OF TRUTH-FUNCTIONAL LOGIC

- 26 Axiomatic System of Truth-Functional Logic, 125
 - 26.0 Primitive Basis of System P, 125
 - 26.1 Basic Concepts of Axiomatics, 127
- 27 Exercises, 130
- 28 Metatheory of System P (I), 132
 - 28.0 Consistency of System P, 132
 - 28.1 Independence of Axioms and Rules, 134
 - 28.2 Independence and Consistency, 140
- 29 Exercises, 141
- 30 Metatheory of System P (II), 144
 - 30.0 The Deduction Theorem, 144
 - 30.1 Some Key Theorem Schemata, 147
 - 30.2 Maximal Consistent Classes, 149
 - 30.3 Completeness Theorem, 151
 - 30.4 Compactness Theorem and Concluding Remarks, 154
- 31 Exercises, 157

PART THREE. SENTENTIAL MODAL LOGIC

- 32 Truth Tables and Modal Logic, 163
 - 32.0 Motivation, 163
 - 32.1 Actual and Possible Truth-Value Outcomes, 163
 - 32.2 Language Schema M, 164
 - 32.3 Full and Partial Truth Tables, 165
 - 32.4 Fundamental Truth Tables Revisited, 169
- 33 Exercises, 173
- 34 Validity in LSM, 175

x Contents

34.0	Value Assignments; Plenary Sets of Truth Tables,	175
34.1	Validity, 179	110

34.2 Relation of LSP to LSM, 179

34.3 Analytic Truth, Logical Truth, Entailment, Implication, and Equivalence, 180

35 Exercises, 181

36 Truth-Tabular Connectives, 183

36.0 Possibility, 183

36.1 The Singulary Truth-Tabular Connectives, 185

36.2 N-ary Truth-Tabular Connectives; Strict Implication, 188

36.3 Strict Equivalence; Compatibility; the Star, 191

37 Exercises, 195

38 Functional Completeness; Reduction of Modal Wffs, 197

38.0 Functional Completeness, 197

38.1 Reduction of Modal Wffs, 200

38.2 The Six Modalities, 201

39 Exercises, 204

40 Axiomatic Modal Logic, 205

40.0 Primitive Basis of the System S5, 205

40.1 Relationship to System P; Consistency of S5, 207

40.2 Deduction Theorem; Key Theorem Schemata, 208

40.3 Completeness Theorem for S5, 211

40.4 The System S5' (Completeness), 216

40.5 The System S5' (Consistency), 218

41 Exercises, 220

PART FOUR. QUANTIFICATION THEORY

- 42 Atomic Analysis, 225
 - 42.0 Molecular Analysis versus Atomic Analysis, 225

42.1 Singular Terms, 225

42.2 Predicates and Circled Numerals, 226

42.3 Transparent versus Opaque Predicates, 228

42.4 Individual Variables and Predicate Variables, 230

43 Exercises, 231

44 Semantics of Atomic Wffs, 232

- 44.0 Semantics of Individual Variables, 232
- 44.1 Semantics of Predicate Variables, 233
- 44.2 Semantics of Atomic Wffs, 234
- 45 Exercises, 236
- 46 Quantifiers, 237
 - 46.0 Existential and Universal Quantifiers, 237
 - 46.1 Grammar of LSQ, 238
 - 46.2 Free and Bound Variables, 239
 - 46.3 Interpretations; Minimal Interpretations, 240
 - 46.4 Inductive Definition of the Value of a Wff Under a Minimal Interpretation, 242
 - 46.5 Applying the Inductive Definition, 243
- 47 Exercises, 245
- 48 Model Theory, 248
 - 48.0 Models: Satisfiability and Validity, 248
 - 48.1 Inflation Theorem, 249
 - 48.2 Löwenheim Theorem; Spectrum Problem, 251
 - 48.3 Generalized Inflation Theorem; Löwenheim-Skolem Theorem, 252
 - 48.4 Implication and Truth-Functional Implication; Equivalence, 254
- 49 Exercises, 255
- 50 Logical Analysis of English Discourse, 259
 - 50.0 Logical Truth and the Empty Domain, 259
 - 50.1 Universes of Discourse of English Statements, 261
 - 50.2 Translating English into Q-Languages, 262
- 51 Exercises, 268
- 52 Quine's System of Natural Deduction (I), 273
 - 52.0 Instances, 273
 - 52.1 Natural Deduction Systems versus Logistic Systems, 274
 - 52.2 Rule of Premiss, 275
 - 52.3 Rule of Truth Functions, 276
 - 52.4 Rule of Universal Instantiation, 277
 - 52.5 Rule of Existential Generalization, 278
 - 52.6 Rule of Conditionalization, 278
 - 52.7 The Five Soundness-Preserving Rules, 280
- 53 Exercises, 281

54	Quir	ne's System of Natural Deduction (II), 284	
	54.0	Conservative Instances, 284	
	54.1	Control Control (11 20)	
	54.2	Rule of Existential Instantiation, 287	
	54.3	Rationale Behind Universal Generalization and Existential Instantiation, 288	
	54.4	Finished Deductions; Proofs; Metatheorems 289	
	54.5	Proof of the Consistency Theorem, 293	
55	Exercises, 296		
56	Appl	ying the Natural Deduction System, 300	
	56.0	Deductive Strategies, 300	
	56.1		
	56.2	Identity, 304	
	56.3	Postulate Systems; Calculus of Individuals, 307	
	90.4	Completeness Theorem for System QI, 313	
57		cises, 316	
58	Proof	f of the Completeness Theorem for System Q, 320	
	99.0	Corollaries of the Skolem-Gödel Theorem 320	
	98.1	Maximal, Consistent, ω-Complete Classes 221	
	58.2	Proof of the Lemma of Section 58.0, 323	
59	Quan	tification with Function Variables, 325	
	59.0	Function Variables: Terms, 325	
	59.1	Natural Deduction Rules for System QIF, 327	
00	39.Z	reano Arithmetic, 328	
60		ises, 333	
61	Decisi	ion Problems and Incompleteness, 335	
	01.0	Decidability: Church's Thesis 225	
	61.1	Church's Theorem, 338	
00	61.2	1100mpicteriess Theorem, 341	
62	Specia	d Cases of the Decision Problem, 343	
	02.0	Special Cases, 343	
	62.1	Syllogisms, 344	
	62.2	Reduction of Decision Problems; Prenex Normal	
eo		roms, 348	
63	Exerci	ses, 350	

APPENDIXES

Appendix A. Set Theory, 355 Introduction to Set Theory, 355 Exercises, 357

Appendix B. Semantic Tableaux, 358

Semantic Tableaux for Truth-Functional Logic, 358

Exercises, 364

Semantic Tableaux for Quantificational Logic, 365

Exercises, 369

Appendix C. Alternative Proof of the Completeness of System P, 372

Exercises, 374

Appendix D. Alternative Proof of the Compactness Theorem for System P, 376

Exercises, 378

Appendix E. Alternative Proof of the Completeness of System Q, 379

Exercises, 383

Appendix F. Alternative Approaches to the Semantics of Quantifiers, 384

Exercises, 386

Appendix G. Quantification Theory with Modality, 389

Kripke's 1959 Semantics; LSQ-M, 389

System Q-M; Natural Deduction System of Quantification Theory with Modality, 392

Alternative Semantics for Quantification Theory with Modality (KA), 397

Alternative Semantics for Quantification Theory with Modality (KB), 398

Exercises, 401

Appendix H. Tense Logic, 404

Exercises, 411

Appendix I. Logistic System of Quantification Theory, 414 Exercises, 415

Index, 417