## TABLE OF CONTENTS

INTRODUCTION by Wolfgang Edelstein and Wolfgang Lefèvre	хi
ON ACTION AND COGNITION	
CHAPTER 1. ACTION AND COGNITION IN PIAGET'S GENETIC EPISTEMOLOGY AND IN HEGEL'S LOGIC	1
Notes Bibliography	10 27
CHAPTER 2. REPRESENTATION AND MEANING	29
Examples of Cognitive Structures Cognitive Structures and Material Action Cognitive Structures and Language Notes Bibliography	29 42 46 54 67
EDUCATION IN CONTEXT	
CHAPTER 3. PHILOSOPHICAL AND PEDAGOGICAL REMARKS ON THE CONCEPT "ABSTRACT"	71
The Common Usage of the Concept in the Didactics of Mathematics Abstraction as the Isolation of Qualities The Concept of Abstraction of Formal Logic The Empiricist Concept of Abstraction Models of Abstraction Based on Constitution Theory	71 72 73 75 75
The Dialectical Concept of Abstraction The "Normal" Pattern of Teaching On the Interpretation of Learning Processes in Cognitive Psychology Understanding the Facts	76 78 79 80

The Function of Exercises	81
Abstraction and Concretization in Mathematics Teaching	83
Bibliography	86
CHAPTER 4. WHAT IS MATHEMATICAL ABILITY AND HOW	
DO ABILITY DIFFERENCES EMERGE IN MATHEMATICS	87
EDUCATION?	0/
The Appearance of Mathematical Competence in Mathematics	
Education	87
Psychometric Constructs of Ability	93
Abilities as Cognitive Structures	98
Notes	108
Bibliography	110
CHAPTER 5. MATHEMATICS EDUCATION AND SOCIETY	111
Introduction	111
The Vicinities of Mathematics— Mathematics of the Vicinities	113
Transmission of Knowledge Predetermined By Specific Ends	118
The Training of the Mind	122
Mathematics as a Profession	126
The Emergence of School Mathematics	131
Mathematics Education in the Context of Modern Institutional	
Conditions	138
Notes	144
Bibliography	145
CHAPTER 6. PRELIMINARY REMARKS ON THE RELATIONSHIP	
OF THE PRINCIPLES OF TEACHING ARITHMETIC TO THE EARLY	4.40
HISTORY OFMATHEMATICS	149
In Retrospect: Repetition in Unison	149
Action as the Starting Point for Mathematical Thinking?	151
On the Role of Historical Conditions of the Development of	
Mathematical Thinking	152
The Early History of Arithmetic as a Touchstone	153
Arithmetic as a System of Rules for Material	154
Representatives of Numbers	154
The Arithmetic of Constructive-Additive Sign Systems	156
Developmental Stages of Mesopotamian Arithmetic	159
	-

Theoretical Perspective	167	
Notes Bibliography	169 170	
CULTURAL EVOLUTION OF ARITHMETICAL THINKING		
IIIIIVKIIVO		
CHAPTER 7. THE DEVELOPMENT OF ARITHMETICAL THINKING: ON THE ROLE OF CALCULATING AIDS IN ANCIENT EGYPTIAN AND BABYLONIAN ARITHMETIC	173	
Preliminary Remarks: Reckoning Board and Rod Numerals	173	
Structural Characteristics of Ancient Egyptian Arithmetic	176	
The Means of Calculation in Ancient Egypt	188	
The Sources for Reconstructing the History of Old Babylonian		
Arithmetic	199	
Structural Characteristics of Old Babylonian Arithmetic	204	
The History of Mesopotamian Calculating Aids	211	
CHAPTER 8. THE FIRST REPRESENTATIONS OF NUMBERS		
AND THE DEVELOPMENT OF THE NUMBER CONCEPT	275	
Characteristics of the Numerical Signs in the Archaic Texts	275	
Earlier Attempts at Interpretation	276	
Rules for Using the Signs	279	
Summations	281	
Statistics as a Method of Decipherment	284	
The Decipherment of Calendar Entries	286	
Numerical Sign Systems with Specific Areas of Application	288	
The Lack of an Abstract Number Concept	291	
Number Analogues	293 294	
From Number Analogues to the Abstract Number Concept	294	
Bibliography	291	
CHAPTER 9. ON THE RELATIONSHIP BETWEEN ONTOGENESIS	200	
AND HISTORIOGENESIS OF THE NUMBER CONCEPT	299	
On the Preconditions of Piaget's Constructivist Conception of Numbers	301	
NUMBER	201	

TABLE OF CONTENTS

ix

Ontogenesis of the Number Concept in Piaget's Genetic	
Epistemology	302
Historiogenesis of the Number Concept in Piaget's Genetic	
Epistemology	308
Symbolic Representation and Historical Transmission	314
Protoarithmetic of a Stone Age Culture	321
Protoarithmetical Techniques in the Transition to a Literate	
Culture	329
Ontogenesis and Historiogenesis of the Number Concept	354
Notes	362
Bibliography	367
ON HISTORICAL EPISTEMOLOGY	
CHAPTER 10. ABSTRACTION AND REPRESENTATION	371
Bibliography	381
CHAPTER 11. THE CONCEPT OF LABOR IN HISTORICAL	
MATERIALISM AND THE THEORY OF SOCIO-HISTORICAL	
DEVELOPMENT	383
Labor in Human History	383
Biological Evolution and Historical Development	387
CHAPTER 12. TOOLS OF SCIENCE	395
Labor and Cognition	395
The Emergence of Science	396
The Role of the Tools of Scientific Work	398
Continuity and Discontinuity of the Development of Science	401
LIST OF ORIGINAL PUBLICATIONS	405
NAME INDEX	407
SUBJECT INDEX	411