
Contents

List of Symbols	xix
List of Acronyms	xxi
1 Introduction	1
1.1 Computational Electromagnetics	2
1.2 Maxwell's Equations	3
1.2.1 Boundary Conditions	4
1.2.2 Energy Relations	5
1.2.3 Time Evolution	6
1.2.4 Dispersion Relation and Wave Velocities	7
1.2.5 Low-Frequency Approximation	8
1.2.6 Integral Formulation	9
2 Convergence	11
2.1 Extrapolation to Zero Cell Size	12
2.1.1 A Singular Problem	15
2.2 Practical Procedures	16
3 Finite Differences	19
3.1 A 2D Capacitance Problem	20
3.1.1 Iterative Solution of Laplace's Equation	21
3.1.2 Computing the Capacitance	22
3.1.3 MATLAB: Capacitance of Coaxial Cable	23
3.2 Finite Difference Derivatives of Complex Exponentials	27
3.2.1 First-Order Derivative	27
3.2.2 Spurious Solutions and Staggered Grids	28
3.2.3 Second-Order Derivative	30

4	Eigenvalues	37
4.1	Maxwell's Equations	37
4.2	Model Problems	38
4.3	Frequency-Domain Eigenvalue Calculation	39
4.3.1	MATLAB: The 1D Helmholtz Equation	39
4.4	Time-Domain Eigenvalue Calculation	42
4.4.1	Stability Analysis	43
4.4.2	MATLAB: The 1D Wave Equation	45
4.4.3	Extracting the Eigenfrequencies	47
4.4.4	MATLAB: Padé Approximation	50
5	The Finite-Difference Time-Domain Method	57
5.1	The 1D Wave Equation	58
5.1.1	Dispersion and Stability	59
5.2	The FDTD Method: Staggered Grids	63
5.2.1	One Space Dimension	63
5.2.2	Three Space Dimensions	65
5.2.3	MATLAB: Cubical Cavity	68
5.2.4	Integral Interpretation of the FDTD Method	72
5.2.5	Dispersion Analysis in Three Dimensions	75
5.3	Boundary Conditions for Open Regions	79
5.3.1	The Perfectly Matched Layer	79
5.3.2	Near-to-Far-Field Transformation	81
6	The Finite Element Method	87
6.1	General Recipe	88
6.2	1D Finite Element Analysis	89
6.3	2D Finite Element Analysis	92
6.3.1	The Assembling Procedure	95
6.3.2	Unstructured Meshes in Practice	99
6.3.3	MATLAB: 2D FEM Using Nodal Basis Functions	100
6.4	Adaptivity	104
6.5	Vector Equations	106
6.5.1	Mixed-Order FEM for Systems of First-Order Equations	106
6.5.2	The Curl-Curl Equation and Edge Elements	108
6.5.3	Edge Elements on Cartesian Grids	110
6.5.4	Eigenfrequencies of a Rectangular Cavity	113
6.5.5	Edge Elements on Triangles	121
6.5.6	Edge Elements in Practice	123
6.5.7	MATLAB: FEM with Triangular Edge Elements	123
6.5.8	Time-Dependent Problems	130
6.6	Magnetostatics and Eddy Current Problems	132
6.6.1	2D Formulation	133
6.6.2	A 2D Application Problem	133

6.6.3	3D Eddy Current Calculations	136
6.7	Variational Methods	139
6.7.1	Relation Between Linear Differential Equations and Quadratic Forms	139
6.7.2	Rayleigh–Ritz Method	143
6.7.3	Galerkin’s Method	143
6.7.4	A Variational Method for Maxwell’s Equations	144
7	The Method of Moments	153
7.1	Integral Formulation of Electrostatics	153
7.1.1	Green’s Function	154
7.1.2	General Formulation	156
7.1.3	FEM Solution	157
7.2	Capacitance Problem in an Unbounded 2D Region	159
7.2.1	Integration	160
7.2.2	MATLAB: MoM for General, 2D Geometries	161
7.2.3	Charge Distribution	165
7.2.4	Adaptivity	166
7.2.5	Numerical Integration	168
7.3	Electromagnetic Scattering	170
7.3.1	Representation by Potentials and a Lorentz Gauge	170
7.3.2	Green’s Function for the Vector Potential	171
7.3.3	The Electric Field Integral Equation	173
7.3.4	The Magnetic Field Integral Equation	176
7.3.5	The Combined Field Integral Equation	177
7.4	Scattering on Thin Wires	178
7.4.1	Hallén’s Equation	179
7.4.2	Valid Approximation for the 1D Kernel	180
7.4.3	Numerical Solution	181
7.4.4	MATLAB: Hallén’s Equation	181
7.4.5	Numerical Results	184
8	Summary and Overview	191
8.1	Differential Equation Solvers	192
8.1.1	Finite-Difference Time-Domain	193
8.1.2	Finite-Volume Time-Domain	193
8.1.3	Finite Element Method	194
8.1.4	Transmission Line Method	194
8.1.5	Finite Integration Technique	195
8.2	Integral Equation Solvers	195
8.2.1	Frequency-Domain Integral Equations	196
8.2.2	Time-Domain Integral Equations	197
8.3	Hybrid Methods	198

A Large Linear Systems	201
A.1 Sparse Matrices	201
A.2 Solvers for Large Sparse Systems of Equations	204
A.2.1 Direct Solvers	204
A.2.2 Iterative Solvers	204
A.2.3 Multigrid Methods	205
A.3 Capacitance Calculation on Larger Grids	206
B Krylov Methods	209
B.1 Projection Methods	209
B.2 Krylov Methods	210
B.3 Nonsymmetric A	211
B.4 Preconditioning	212
References	213
Index	219