## Contents

	Preface	xvii	
I	THE SETTING: MARKETS, MODELS, INTER UTILITY MAXIMIZATION, RISK	REST RATES,	
1	Financial Markets	3	
1.1	Bonds	3	
	1.1.1 Types of Bonds	5	
	1.1.2 Reasons for Trading Bonds	5	
	1.1.3 Risk of Trading Bonds	6	
1.2		7	
	1.2.1 How Are Stocks Different from Bonds?	8	
	1.2.2 Going Long or Short	Ģ	
1.3	Derivatives	ģ	
	1.3.1 Futures and Forwards	1(	
	1.3.2 Marking to Market	11	
	1.3.3 Reasons for Trading Futures	12	
	1.3.4 Options	13	
	1.3.5 Calls and Puts	13	
	1.3.6 Option Prices	15	
	1.3.7 Reasons for Trading Options	16	
	1.3.8 Swaps	17	
	1.3.9 Mortgage-Backed Securities; Callable Bonds	19	
1.4	8	20	
	1.4.1 Exchanges	20	
	1.4.2 Market Indexes	21	
1.5		22	
	1.5.1 Trades That Involve Margin Requirements	23	
1.6		24 25	
	Summary		
	Problems		
	Further Readings	29	
2	Interest Rates	3	
2.1	Computation of Interest Rates		
	2.1.1 Simple versus Compound Interest; Annualize	d Rates 32	
	2.1.2 Continuous Interest	3,	

x Contents

	4.6.2	Incomplete Information in Continuous Time*	140
	4.6.3	Power Utility and Normally Distributed Drift*	142
4.7	Apper	ndix: Proof of Dynamic Programming Principle	145
	Summ	nary	146
	Proble	ems	147
	Furthe	er Readings	150
5	Risk		153
5.1	Risk v	versus Return: Mean-Variance Analysis	153
	5.1.1	Mean and Variance of a Portfolio	154
	5.1.2	Mean-Variance Efficient Frontier	157
	5.1.3	Computing the Optimal Mean-Variance Portfolio	160
	5.1.4	Computing the Optimal Mutual Fund	163
	5.1.5	Mean-Variance Optimization in Continuous Time*	164
5.2	VaR:	Value at Risk	167
	5.2.1	Definition of VaR	167
	5.2.2	Computing VaR	168
	5.2.3	VaR of a Portfolio of Assets	170
	5.2.4	Alternatives to VaR	171
	5.2.5	The Story of Long-Term Capital Management	171
	Sumn	nary	172
	Proble	ems	172
	Furth	er Readings	175
II	PRIC	ING AND HEDGING OF DERIVATIVE SECURITIES	177
6	Arbit	rage and Risk-Neutral Pricing	179
6.1		rage Relationships for Call and Put Options; Put-Call Parity	179
6.2		rage Pricing of Forwards and Futures	184
	6.2.1	Forward Prices	184
	6.2.2	Futures Prices	186
	6.2.3	Futures on Commodities	187
6.3	Risk-	Neutral Pricing	188
	6.3.1	Martingale Measures; Cox-Ross-Rubinstein (CRR) Model	188
	6.3.2	State Prices in Single-Period Models	192
	633	No Arbitrage and Risk-Neutral Probabilities	107

Contents xi

	6.3.4	Pricing by No Arbitrage	194	
	6.3.5	Pricing by Risk-Neutral Expected Values	196	
	6.3.6	Martingale Measure for the Merton-Black-Scholes Model	197	
	6.3.7	Computing Expectations by the Feynman-Kac PDE	201	
	6.3.8	Risk-Neutral Pricing in Continuous Time	202	
	6.3.9	Futures and Forwards Revisited*	203	
6.4	Apper	ndix	206	
	6.4.1	No Arbitrage Implies Existence of a Risk-Neutral Probability*	206	
	6.4.2	Completeness and Unique EMM*	207	
	6.4.3	Another Proof of Theorem 6.4*	210	
	6.4.4	Proof of Bayes' Rule**	211	
	Sumn	nary	211	
	Proble	ems	213	
	Furthe	er Readings	215	
7	Optio	n Pricing	217	
7.1	Optio	n Pricing in the Binomial Model	217	
	7.1.1	Backward Induction and Expectation Formula	217	
	7.1.2	Black-Scholes Formula as a Limit of the Binomial		
		Model Formula	220	
7.2	_	Option Pricing in the Merton-Black-Scholes Model		
	7.2.1	Black-Scholes Formula as Expected Value	222	
	7.2.2	Black-Scholes Equation	222	
	7.2.3	Black-Scholes Formula for the Call Option	225	
	7.2.4	Implied Volatility	227	
7.3	Pricin	g American Options	228	
	7.3.1	Stopping Times and American Options	229	
	7.3.2	Binomial Trees and American Options	231	
	7.3.3	PDEs and American Options*	233	
7.4	Optio	Options on Dividend-Paying Securities		
	7.4.1	Binomial Model	236	
	7.4.2	Merton-Black-Scholes Model	238	
7.5	Other	Types of Options	240	
	7.5.1	Currency Options	240	
	7.5.2	Futures Options	242	
	7.5.3	Exotic Options	243	

xii Contents

7.6	Pricin	g in the Presence of Several Random Variables	247
	7.6.1	Options on Two Risky Assets	248
	7.6.2	Quantos	252
	7.6.3	Stochastic Volatility with Complete Markets	255
	7.6.4	Stochastic Volatility with Incomplete Markets; Market Price	
		of Risk*	256
	7.6.5	Utility Pricing in Incomplete Markets*	257
7.7	Merto	on's Jump-Diffusion Model*	260
7.8	Estim	ation of Variance and ARCH/GARCH Models	262
7.9	Apper	ndix: Derivation of the Black-Scholes Formula	265
	Sumn	nary	267
	Proble	ems	268
	Furthe	er Readings	273
8	Fixed	-Income Market Models and Derivatives	275
8.1	Discre	ete-Time Interest-Rate Modeling	275
	8.1.1	Binomial Tree for the Interest Rate	276
	8.1.2	Black-Derman-Toy Model	279
	8.1.3	Ho-Lee Model	281
8.2	Intere	st-Rate Models in Continuous Time	286
	8.2.1	One-Factor Short-Rate Models	287
	8.2.2	Bond Pricing in Affine Models	289
	8.2.3	HJM Forward-Rate Models	291
	8.2.4	Change of Numeraire*	295
	8.2.5	Option Pricing with Random Interest Rate*	296
	8.2.6	BGM Market Model*	299
8.3	Swaps, Caps, and Floors		
	8.3.1	Interest-Rate Swaps and Swaptions	301
	8.3.2	Caplets, Caps, and Floors	305
8.4	Credit/Default Risk		
	Summ	nary	308
	Proble	ems	309
	Further Readings		312
9	Hedgi	ing	313
9.1	Hedgi	ng with Futures	313
	9.1.1	Perfect Hedge	313

Contents

	9.1.2	Cross-Hedging; Basis Risk	314
	9.1.3	Rolling the Hedge Forward	316
	9.1.4	Quantity Uncertainty	317
9.2	Portfo	lios of Options as Trading Strategies	317
	9.2.1	Covered Calls and Protective Puts	318
	9.2.2	Bull Spreads and Bear Spreads	318
	9.2.3	Butterfly Spreads	319
	9.2.4	Straddles and Strangles	321
9.3	Hedgi	322	
	9.3.1	Delta Hedging in Discrete-Time Models	323
	9.3.2	Delta-Neutral Strategies	325
	9.3.3	Deltas of Calls and Puts	327
	9.3.4	Example: Hedging a Call Option	327
	9.3.5	Other Greeks	330
	9.3.6	Stochastic Volatility and Interest Rate	332
	9.3.7	Formulas for Greeks	333
	9.3.8	Portfolio Insurance	333
9.4	Perfec	t Hedging in a Multivariable Continuous-Time Model	334
9.5	Hedgi	ng in Incomplete Markets	335
	Summ	ary	336
	Proble	ms	337
	Furthe	r Readings	340
10	Bond	Hedging	341
10.1	Durati	341	
	10.1.1	Definition and Interpretation	341
	10.1.2	Duration and Change in Yield	345
	10.1.3	Duration of a Portfolio of Bonds	346
10.2	Immunization		347
	10.2.1	Matching Durations	347
	10.2.2	Duration and Immunization in Continuous Time	350
10.3	Convexity		351
	Summ	352	
	Proble	352	
	Furthe	353	

xiv Contents

11	Numer	rical Methods	355
11.1	Binomial Tree Methods		
	11.1.1	Computations in the Cox-Ross-Rubinstein Model	355
	11.1.2	Computing Option Sensitivities	358
	11.1.3	Extensions of the Tree Method	359
11.2	Monte	Carlo Simulation	361
	11.2.1	Monte Carlo Basics	362
	11.2.2	Generating Random Numbers	363
	11.2.3	Variance Reduction Techniques	364
	11.2.4	Simulation in a Continuous-Time Multivariable Model	367
	11.2.5	Computation of Hedging Portfolios by Finite Differences	370
	11.2.6	Retrieval of Volatility Method for Hedging and	
		Utility Maximization*	371
11.3	Numer	ical Solutions of PDEs; Finite-Difference Methods	373
	11.3.1	Implicit Finite-Difference Method	374
	11.3.2	Explicit Finite-Difference Method	376
	Summary		377
	Problems		378
	Further	Readings	380
Ш	EQUII	LIBRIUM MODELS	381
12	Equilil	orium Fundamentals	383
12.1	Concer	ot of Equilibrium	383
	12.1.1	Definition and Single-Period Case	383
	12.1.2	A Two-Period Example	387
	12.1.3	Continuous-Time Equilibrium	389
12.2	Single-Agent and Multiagent Equilibrium		
	12.2.1	Representative Agent	389
	12.2.2	Single-Period Aggregation	389
12.3	Pure E	schange Equilibrium	391
	12.3.1	Basic Idea and Single-Period Case	392
	12.3.2	Multiperiod Discrete-Time Model	394
	12.3.3	Continuous-Time Pure Exchange Equilibrium	395
12.4	Existence of Equilibrium		398
	12.4.1	Equilibrium Existence in Discrete Time	399

Contents

	12.4.2	Equilibrium Existence in Continuous Time	400
	12.4.3	Determining Market Parameters in Equilibrium	403
	Summa	ry	406
	Problem	ns	406
	Further	Readings	407
13	CAPM		409
13.1	Basic C	CAPM	409
	13.1.1	CAPM Equilibrium Argument	409
	13.1.2	Capital Market Line	411
	13.1.3	CAPM formula	412
13.2	Econon	nic Interpretations	413
	13.2.1	Securities Market Line	413
	13.2.2	Systematic and Nonsystematic Risk	414
	13.2.3	Asset Pricing Implications: Performance Evaluation	416
	13.2.4	Pricing Formulas	418
	13.2.5	Empirical Tests	419
13.3	Alterna	tive Derivation of the CAPM*	420
13.4	Continu	ious-Time, Intertemporal CAPM*	423
13.5	Consun	nption CAPM*	427
	Summa	ry	430
	Problen	ns	430
	Further	Readings	432
14	Multifa	actor Models	433
14.1	Discrete	e-Time Multifactor Models	433
14.2	Arbitra	ge Pricing Theory (APT)	436
14.3	Multifa	ctor Models in Continuous Time*	438
	14.3.1	Model Parameters and Variables	438
	14.3.2	Value Function and Optimal Portfolio	439
	14.3.3	Separation Theorem	441
	14.3.4	Intertemporal Multifactor CAPM	442
	Summa	ry	445
	Problems		445
	Further	Readings	445

xvi Contents

15	Other :	Pure Exchange Equilibria	447
15.1	Term-Structure Equilibria		
	15.1.1	Equilibrium Term Structure in Discrete Time	447
	15.1.2	Equilibrium Term Structure in Continuous Time; CIR Model	449
15.2	Informa	ational Equilibria	451
	15.2.1	Discrete-Time Models with Incomplete Information	451
	15.2.2	Continuous-Time Models with Incomplete Information	454
15.3	Equilib	rium with Heterogeneous Agents	457
	15.3.1	Discrete-Time Equilibrium with Heterogeneous Agents	458
	15.3.2	Continuous-Time Equilibrium with Heterogeneous Agents	459
15.4	Interna	tional Equilibrium; Equilibrium with Two Prices	461
	15.4.1	Discrete-Time International Equilibrium	462
	15.4.2	Continuous-Time International Equilibrium	463
	Summary		466
	Problems		466
	Further	Readings	467
16	Appen	dix: Probability Theory Essentials	469
16.1	Discret	e Random Variables	469
	16.1.1	Expectation and Variance	469
16.2	Continu	uous Random Variables	470
	16.2.1	Expectation and Variance	470
16.3	Several	Random Variables	471
	16.3.1	Independence	471
	16.3.2	Correlation and Covariance	472
16.4	Normal	Random Variables	472
16.5	Propert	ies of Conditional Expectations	474
16.6	Marting	gale Definition	476
16.7	Randor	n Walk and Brownian Motion	476
	Referen	nces	479
	Index		487