

Contents

Preface	vii
I Fundamentals of MDS	1
1 The Four Purposes of Multidimensional Scaling	3
1.1 MDS as an Exploratory Technique	4
1.2 MDS for Testing Structural Hypotheses	6
1.3 MDS for Exploring Psychological Structures	9
1.4 MDS as a Model of Similarity Judgments	11
1.5 The Different Roots of MDS	13
1.6 Exercises	15
2 Constructing MDS Representations	19
2.1 Constructing Ratio MDS Solutions	19
2.2 Constructing Ordinal MDS Solutions	23
2.3 Comparing Ordinal and Ratio MDS Solutions	29
2.4 On Flat and Curved Geometries	30
2.5 General Properties of Distance Representations	33
2.6 Exercises	34
3 MDS Models and Measures of Fit	37
3.1 Basics of MDS Models	37
3.2 Errors, Loss Functions, and Stress	41

3.3	Stress Diagrams	42
3.4	Stress per Point	44
3.5	Evaluating Stress	47
3.6	Recovering True Distances by Metric MDS	55
3.7	Further Variants of MDS Models	57
3.8	Exercises	59
4	Three Applications of MDS	63
4.1	The Circular Structure of Color Similarities	63
4.2	The Regionality of Morse Codes Confusions	68
4.3	Dimensions of Facial Expressions	73
4.4	General Principles of Interpreting MDS Solutions	80
4.5	Exercises	82
5	MDS and Facet Theory	87
5.1	Facets and Regions in MDS Space	87
5.2	Regional Laws	91
5.3	Multiple Facetizations	93
5.4	Partitioning MDS Spaces Using Facet Diagrams	95
5.5	Prototypical Roles of Facets	99
5.6	Criteria for Choosing Regions	100
5.7	Regions and Theory Construction	102
5.8	Regions, Clusters, and Factors	104
5.9	Exercises	105
6	How to Obtain Proximities	111
6.1	Types of Proximities	111
6.2	Collecting Direct Proximities	112
6.3	Deriving Proximities by Aggregating over Other Measures	119
6.4	Proximities from Converting Other Measures	125
6.5	Proximities from Co-Occurrence Data	126
6.6	Choosing a Particular Proximity	128
6.7	Exercises	130
II	MDS Models and Solving MDS Problems	135
7	Matrix Algebra for MDS	137
7.1	Elementary Matrix Operations	137
7.2	Scalar Functions of Vectors and Matrices	142
7.3	Computing Distances Using Matrix Algebra	144
7.4	Eigendecompositions	146
7.5	Singular Value Decompositions	150
7.6	Some Further Remarks on SVD	152
7.7	Linear Equation Systems	154

7.8	Computing the Eigendecomposition	157
7.9	Configurations that Represent Scalar Products	160
7.10	Rotations	160
7.11	Exercises	163
8	A Majorization Algorithm for Solving MDS	169
8.1	The Stress Function for MDS	169
8.2	Mathematical Excursus: Differentiation	171
8.3	Partial Derivatives and Matrix Traces	176
8.4	Minimizing a Function by Iterative Majorization	178
8.5	Visualizing the Majorization Algorithm for MDS	184
8.6	Majorizing Stress	185
8.7	Exercises	194
9	Metric and Nonmetric MDS	199
9.1	Allowing for Transformations of the Proximities	199
9.2	Monotone Regression	205
9.3	The Geometry of Monotone Regression	209
9.4	Tied Data in Ordinal MDS	211
9.5	Rank-Images	213
9.6	Monotone Splines	214
9.7	A Priori Transformations Versus Optimal Transformations	221
9.8	Exercises	224
10	Confirmatory MDS	227
10.1	Blind Loss Functions	227
10.2	Theory-Compatible MDS: An Example	228
10.3	Imposing External Constraints on MDS Representations	230
10.4	Weakly Constrained MDS	237
10.5	General Comments on Confirmatory MDS	242
10.6	Exercises	244
11	MDS Fit Measures, Their Relations, and Some Algorithms	247
11.1	Normalized Stress and Raw Stress	247
11.2	Other Fit Measures and Recent Algorithms	250
11.3	Using Weights in MDS	254
11.4	Exercises	258
12	Classical Scaling	261
12.1	Finding Coordinates in Classical Scaling	261
12.2	A Numerical Example for Classical Scaling	263
12.3	Choosing a Different Origin	264
12.4	Advanced Topics	265
12.5	Exercises	267

13 Special Solutions, Degeneracies, and Local Minima	269
13.1 A Degenerate Solution in Ordinal MDS	269
13.2 Avoiding Degenerate Solutions	272
13.3 Special Solutions: Almost Equal Dissimilarities	274
13.4 Local Minima	276
13.5 Unidimensional Scaling	278
13.6 Full-Dimensional Scaling	281
13.7 The Tunneling Method for Avoiding Local Minima	283
13.8 Distance Smoothing for Avoiding Local Minima	284
13.9 Exercises	288
III Unfolding	291
14 Unfolding	293
14.1 The Ideal-Point Model	293
14.2 A Majorizing Algorithm for Unfolding	297
14.3 Unconditional Versus Conditional Unfolding	299
14.4 Trivial Unfolding Solutions and σ_2	301
14.5 Isotonic Regions and Indeterminacies	305
14.6 Unfolding Degeneracies in Practice and Metric Unfolding	308
14.7 Dimensions in Multidimensional Unfolding	312
14.8 Multiple Versus Multidimensional Unfolding	313
14.9 Concluding Remarks	314
14.10 Exercises	314
15 Avoiding Trivial Solutions in Unfolding	317
15.1 Adjusting the Unfolding Data	317
15.2 Adjusting the Transformation	322
15.3 Adjustments to the Loss Function	324
15.4 Summary	330
15.5 Exercises	331
16 Special Unfolding Models	335
16.1 External Unfolding	335
16.2 The Vector Model of Unfolding	336
16.3 Weighted Unfolding	342
16.4 Value Scales and Distances in Unfolding	345
16.5 Exercises	352
IV MDS Geometry as a Substantive Model	357
17 MDS as a Psychological Model	359
17.1 Physical and Psychological Space	359

17.2	Minkowski Distances	363
17.3	Identifying the True Minkowski Distance	367
17.4	The Psychology of Rectangles	372
17.5	Axiomatic Foundations of Minkowski Spaces	377
17.6	Subadditivity and the MBR Metric	381
17.7	Minkowski Spaces, Metric Spaces, and Psychological Models	385
17.8	Exercises	386
18	Scalar Products and Euclidean Distances	389
18.1	The Scalar Product Function	389
18.2	Collecting Scalar Products Empirically	392
18.3	Scalar Products and Euclidean Distances: Formal Relations	397
18.4	Scalar Products and Euclidean Distances: Empirical Relations	400
18.5	MDS of Scalar Products	403
18.6	Exercises	408
19	Euclidean Embeddings	411
19.1	Distances and Euclidean Distances	411
19.2	Mapping Dissimilarities into Distances	415
19.3	Maximal Dimensionality for Perfect Interval MDS	418
19.4	Mapping Fallible Dissimilarities into Euclidean Distances .	419
19.5	Fitting Dissimilarities into a Euclidean Space	424
19.6	Exercises	425
V	MDS and Related Methods	427
20	Procrustes Procedures	429
20.1	The Problem	429
20.2	Solving the Orthogonal Procrustean Problem	430
20.3	Examples for Orthogonal Procrustean Transformations . . .	432
20.4	Procrustean Similarity Transformations	434
20.5	An Example of Procrustean Similarity Transformations . . .	436
20.6	Configurational Similarity and Correlation Coefficients . . .	437
20.7	Configurational Similarity and Congruence Coefficients . . .	439
20.8	Artificial Target Matrices in Procrustean Analysis	441
20.9	Other Generalizations of Procrustean Analysis	444
20.10	Exercises	445
21	Three-Way Procrustean Models	449
21.1	Generalized Procrustean Analysis	449
21.2	Helm's Color Data	451
21.3	Generalized Procrustean Analysis	454
21.4	Individual Differences Models: Dimension Weights	457

21.5	An Application of the Dimension-Weighting Model	462
21.6	Vector Weightings	465
21.7	PINDIS, a Collection of Procrustean Models	469
21.8	Exercises	471
22	Three-Way MDS Models	473
22.1	The Model: Individual Weights on Fixed Dimensions	473
22.2	The Generalized Euclidean Model	479
22.3	Overview of Three-Way Models in MDS	482
22.4	Some Algebra of Dimension-Weighting Models	485
22.5	Conditional and Unconditional Approaches	489
22.6	On the Dimension-Weighting Models	491
22.7	Exercises	492
23	Modeling Asymmetric Data	495
23.1	Symmetry and Skew-Symmetry	495
23.2	A Simple Model for Skew-Symmetric Data	497
23.3	The Gower Model for Skew-Symmetries	498
23.4	Modeling Skew-Symmetry by Distances	500
23.5	Embedding Skew-Symmetries as Drift Vectors into MDS Plots	502
23.6	Analyzing Asymmetry by Unfolding	503
23.7	The Slide-Vector Model	506
23.8	The Hill-Climbing Model	509
23.9	The Radius-Distance Model	512
23.10	Using Asymmetry Models	514
23.11	Overview	515
23.12	Exercises	515
24	Methods Related to MDS	519
24.1	Principal Component Analysis	519
24.2	Correspondence Analysis	526
24.3	Exercises	537
VI	Appendices	541
A	Computer Programs for MDS	543
A.1	Interactive MDS Programs	544
A.2	MDS Programs with High-Resolution Graphics	550
A.3	MDS Programs without High-Resolution Graphics	562
B	Notation	569
	References	573

Author Index

599

Subject Index

605