

# Contents

<i>List of Figures</i>	xi
<i>List of Tables</i>	xiii
<i>Preface</i>	xv
<i>Acknowledgments</i>	xvii
<b>1 Qualitative Overview</b>	1
1.1 Introduction	1
1.2 Forecasting Mortality	3
1.2.1 The Data	3
1.2.2 The Patterns	5
1.2.3 Scientific versus Optimistic Forecasting Goals	8
1.3 Statistical Modeling	11
1.4 Implications for the Bayesian Modeling Literature	15
1.5 Incorporating Area Studies in Cross-National Comparative Research	16
1.6 Summary	18
<b>Part I Existing Methods for Forecasting Mortality</b>	19
<b>2 Methods without Covariates</b>	21
2.1 Patterns in Mortality Age Profiles	22
2.2 A Unified Statistical Framework	24
2.3 Population Extrapolation Approaches	25
2.4 Parametric Approaches	26
2.5 A Nonparametric Approach: Principal Components	28
2.5.1 Introduction	28
2.5.2 Estimation	32
2.6 The Lee-Carter Approach	34
2.6.1 The Model	34
2.6.2 Estimation	36
2.6.3 Forecasting	36
2.6.4 Properties	38
2.7 Summary	42
<b>3 Methods with Covariates</b>	43
3.1 Equation-by-Equation Maximum Likelihood	43
3.1.1 Poisson Regression	43
3.1.2 Least Squares	44
3.1.3 Computing Forecasts	46
3.1.4 Summary Evaluation	47
3.2 Time-Series, Cross-Sectional Pooling	48
3.2.1 The Model	48
3.2.2 Postestimation Intercept Correction	49
3.2.3 Summary Evaluation	49
3.3 Partially Pooling Cross Sections via Disturbance Correlations	50
3.4 Cause-Specific Methods with Microlevel Information	51

3.4.1	Direct Decomposition Methods	51
	Modeling	51
3.4.2	Microsimulation Methods	52
3.4.3	Interpretation	53
3.5	Summary	53
<b>Part II Statistical Modeling</b>		<b>55</b>
<b>4</b>	<b>The Model</b>	<b>57</b>
4.1	Overview	57
4.2	Priors on Coefficients	59
4.3	Problems with Priors on Coefficients	60
4.3.1	Little Direct Prior Knowledge Exists about Coefficients	61
4.3.2	Normalization Factors Cannot Be Estimated	62
4.3.3	We Know about the Dependent Variable, Not the Coefficients	64
4.3.4	Difficulties with Incomparable Covariates	65
4.4	Priors on the Expected Value of the Dependent Variable	65
4.4.1	Step 1: Specify a Prior for the Dependent Variable	66
4.4.2	Step 2: Translate to a Prior on the Coefficients	67
4.4.3	Interpretation	68
4.5	A Basic Prior for Smoothing over Age Groups	69
4.5.1	Step 1: A Prior for $\mu$	69
4.5.2	Step 2: From the Prior on $\mu$ to the Prior on $\beta$	71
4.5.3	Interpretation	71
4.6	Concluding Remark	73
<b>5</b>	<b>Priors over Grouped Continuous Variables</b>	<b>74</b>
5.1	Definition and Analysis of Prior Indifference	74
5.1.1	A Simple Special Case	76
5.1.2	General Expressions for Prior Indifference	76
5.1.3	Interpretation	77
5.2	Step 1: A Prior for $\mu$	80
5.2.1	Measuring Smoothness	81
5.2.2	Varying the Degree of Smoothness over Age Groups	83
5.2.3	Null Space and Prior Indifference	83
5.2.4	Nonzero Mean Smoothness Functional	85
5.2.5	Discretizing: From Age to Age Groups	85
5.2.6	Interpretation	86
5.3	Step 2: From the Prior on $\mu$ to the Prior on $\beta$	92
5.3.1	Analysis	92
5.3.2	Interpretation	92
<b>6</b>	<b>Model Selection</b>	<b>94</b>
6.1	Choosing the Smoothness Functional	94
6.2	Choosing a Prior for the Smoothing Parameter	97
6.2.1	Smoothness Parameter for a Nonparametric Prior	98
6.2.2	Smoothness Parameter for the Prior over the Coefficients	100
6.3	Choosing Where to Smooth	104

6.4	Choosing Covariates	108
6.4.1	Size of the Null Space	109
6.4.2	Content of the Null Space	110
6.5	Choosing a Likelihood and Variance Function	112
6.5.1	Deriving the Normal Specification	112
6.5.2	Accuracy of the Log-Normal Approximation to the Poisson	114
6.5.3	Variance Specification	120
<b>7</b>	<b>Adding Priors over Time and Space</b>	<b>124</b>
7.1	Smoothing over Time	124
7.1.1	Prior Indifference and the Null Space	125
7.2	Smoothing over Countries	127
7.2.1	Null Space and Prior Indifference	128
7.2.2	Interpretation	130
7.3	Smoothing Simultaneously over Age, Country, and Time	131
7.4	Smoothing Time Trend Interactions	132
7.4.1	Smoothing Trends over Age Groups	133
7.4.2	Smoothing Trends over Countries	133
7.5	Smoothing with General Interactions	134
7.6	Choosing a Prior for Multiple Smoothing Parameters	136
7.6.1	Example	139
7.6.2	Estimating the Expected Value of the Summary Measures	141
7.7	Summary	144
<b>8</b>	<b>Comparisons and Extensions</b>	<b>145</b>
8.1	Priors on Coefficients versus Dependent Variables	145
8.1.1	Defining Distances	145
8.1.2	Conditional Densities	147
8.1.3	Connections to “Virtual Examples” in Pattern Recognition	147
8.2	Extensions to Hierarchical Models and Empirical Bayes	148
8.2.1	The Advantages of Empirical Bayes without Empirical Bayes	149
8.2.2	Hierarchical Models as Special Cases of Spatial Models	151
8.3	Smoothing Data without Forecasting	151
8.4	Priors When the Dependent Variable Changes Meaning	153
<b>Part III</b>	<b>Estimation</b>	<b>159</b>
<b>9</b>	<b>Markov Chain Monte Carlo Estimation</b>	<b>161</b>
9.1	Complete Model Summary	161
9.1.1	Likelihood	162
9.1.2	Prior for $\beta$	162
9.1.3	Prior for $\sigma_i$	162
9.1.4	Prior for $\theta$	163
9.1.5	The Posterior Density	164
9.2	The Gibbs Sampling Algorithm	164
9.2.1	Sampling $\sigma$	165
	The Conditional Density	165
	Interpretation	165

9.2.2	Sampling $\theta$	166
	The Conditional Density	166
	Interpretation	166
9.2.3	Sampling $\beta$	167
	The Conditional Density	167
	Interpretation	168
9.2.4	Uncertainty Estimates	169
9.3	Summary	169
<b>10</b>	<b>Fast Estimation without Markov Chains</b>	<b>170</b>
10.1	Maximum A Posteriori Estimator	170
10.2	Marginal Maximum A Posteriori Estimator	171
10.3	Conditional Maximum A Posteriori Estimator	172
10.4	Summary	173
<b>Part IV</b>	<b>Empirical Evidence</b>	<b>175</b>
<b>11</b>	<b>Illustrative Analyses</b>	<b>177</b>
11.1	Forecasts without Covariates: Linear Trends	178
	11.1.1 Smoothing over Age Groups Only	178
	11.1.2 Smoothing over Age and Time	181
11.2	Forecasts without Covariates: Nonlinear Trends	182
11.3	Forecasts with Covariates: Smoothing over Age and Time	187
11.4	Smoothing over Countries	189
<b>12</b>	<b>Comparative Analyses</b>	<b>196</b>
12.1	All Causes in Males	197
12.2	Lung Disease in Males	200
	12.2.1 Comparison with Least Squares	202
	12.2.2 Country-by-Country Analysis	203
12.3	Breast Cancer in Females	205
	12.3.1 Comparison with Least Squares	205
	12.3.2 Country-by-country Analysis	205
12.4	Comparison on OECD Countries	206
	12.4.1 Transportation Accidents in Males	208
	12.4.2 Cardiovascular Disease in Males	210
<b>13</b>	<b>Concluding Remarks</b>	<b>211</b>
<b>Appendixes</b>		<b>213</b>
<b>A</b>	<b>Notation</b>	<b>215</b>
	A.1 Principles	215
	A.2 Glossary	216
<b>B</b>	<b>Mathematical Refresher</b>	<b>219</b>
	B.1 Real Analysis	219
	B.1.1 Vector Space	219

B.1.2	Metric Space	220
B.1.3	Normed Space	221
B.1.4	Scalar Product Space	222
B.1.5	Functions, Mappings, and Operators	223
B.1.6	Functional	224
B.1.7	Span	224
B.1.8	Basis and Dimension	224
B.1.9	Orthonormality	225
B.1.10	Subspace	225
B.1.11	Orthogonal Complement	226
B.1.12	Direct Sum	226
B.1.13	Projection Operators	227
B.2	Linear Algebra	229
B.2.1	Range, Null Space, Rank, and Nullity	229
B.2.2	Eigenvalues and Eigenvectors for Symmetric Matrices	232
B.2.3	Definiteness	234
B.2.4	Singular Values Decomposition	234
Definition		234
For Approximation		235
B.2.5	Generalized Inverse	236
B.2.6	Quadratic Form Identity	238
B.3	Probability Densities	239
B.3.1	The Normal Distribution	239
B.3.2	The Gamma Distribution	239
B.3.3	The Log-Normal Distribution	240
<b>C</b>	<b>Improper Normal Priors</b>	241
C.1	Definitions	241
C.2	An Intuitive Special Case	242
C.3	The General Case	243
C.4	Drawing Random Samples	246
<b>D</b>	<b>Discretization of the Derivative Operator</b>	247
<b>E</b>	<b>Smoothness over Graphs</b>	249
	<i>Bibliography</i>	251
	<i>Index</i>	259