

CONTENTS

About the Author	viii
Series Editor's Introduction	ix
Preface	xi
Notation	xii
Acknowledgments	xiv
1. Matrices, Linear Algebra, and Vector Geometry	1
1.1 Matrices	2
1.1.1 Introducing the Actors: Basic Definitions	2
1.1.2 Simple Matrix Arithmetic	5
1.1.3 Matrix Inverses	11
1.1.4 Determinants	16
1.1.5 The Kronecker Product	16
1.2 Basic Vector Geometry	18
1.3 Vector Spaces and Subspaces	20
1.3.1 Orthogonality and Orthogonal Projections	24
1.4 Matrix Rank and the Solution of Linear Simultaneous Equations	30
1.4.1 Rank	30
1.4.2 Linear Simultaneous Equations	32
1.4.3 Generalized Inverses	37
1.5 Eigenvalues and Eigenvectors	41
1.6 Quadratic Forms and Positive-Definite Matrices	45
1.6.1 The Cholesky Decomposition	46
1.7 Recommended Reading	47
2. An Introduction to Calculus	48
2.1 Review	48
2.1.1 Numbers	48
2.1.2 Lines and Planes	49
2.1.3 Polynomials	51
2.1.4 Logarithms and Exponentials	52
2.1.5 Basic Trigonometric Functions	54
2.2 Limits	55
2.2.1 The “Epsilon-Delta” Definition of a Limit	56
2.2.2 Finding a Limit: An Example	57
2.2.3 Rules for Manipulating Limits	58

2.3	The Derivative of a Function	59
2.3.1	The Derivative as the Limit of the Difference Quotient: An Example	61
2.3.2	Derivatives of Powers	61
2.3.3	Rules for Manipulating Derivatives	62
2.3.4	Derivatives of Logs and Exponentials	65
2.3.5	Derivatives of the Basic Trigonometric Functions	65
2.3.6	Second-Order and Higher-Order Derivatives	66
2.4	Optimization	66
2.4.1	Optimization: An Example	68
2.5	Multivariable and Matrix Differential Calculus	71
2.5.1	Partial Derivatives	71
2.5.2	Lagrange Multipliers for Constrained Optimization	73
2.5.3	Differential Calculus in Matrix Form	74
2.6	Taylor Series	77
2.7	Essential Ideas of Integral Calculus	79
2.7.1	Areas: Definite Integrals	79
2.7.2	Indefinite Integrals	80
2.7.3	The Fundamental Theorem of Calculus	81
2.8	Recommended Reading	83
3.	Probability and Estimation	84
3.1	Elementary Probability Theory	84
3.1.1	Probability Basics	84
3.1.2	Random Variables	89
3.1.3	Transformations of Random Variables	96
3.2	Some Discrete Probability Distributions	98
3.2.1	The Binomial and Bernoulli Distributions	99
3.2.2	The Multinomial Distributions	100
3.2.3	The Poisson Distributions	101
3.2.4	The Negative Binomial Distributions	102
3.3	Some Continuous Distributions	103
3.3.1	The Normal Distributions	103
3.3.2	The Chi-Square (χ^2) Distributions	105
3.3.3	Student's <i>t</i> -Distributions	106
3.3.4	The <i>F</i> -Distributions	107
3.3.5	The Multivariate-Normal Distributions	109
3.3.6	The Exponential Distributions	109
3.3.7	The Inverse-Gaussian Distributions	110
3.3.8	The Gamma Distributions	111
3.3.9	The Beta Distributions	112

3.4	Asymptotic Distribution Theory: An Introduction	113
3.4.1	Probability Limits	113
3.4.2	Asymptotic Expectation and Variance	115
3.4.3	Asymptotic Distribution	117
3.4.4	Vector and Matrix Random Variables	118
3.5	Properties of Estimators	119
3.5.1	Bias	119
3.5.2	Mean-Squared Error and Efficiency	120
3.5.3	Consistency	121
3.5.4	Sufficiency	122
3.5.5	Robustness	123
3.6	Maximum-Likelihood Estimation	131
3.6.1	Preliminary Example	131
3.6.2	Properties of Maximum-Likelihood Estimators	134
3.6.3	Statistical Inference: Wald, Likelihood-Ratio, and Score Tests	135
3.6.4	Several Parameters	139
3.6.5	The Delta Method	143
3.7	Introduction to Bayesian Inference	144
3.7.1	Bayes's Theorem	144
3.7.2	Extending Bayes's Theorem	146
3.7.3	An Example of Bayesian Inference	148
3.7.4	Bayesian Interval Estimates	150
3.7.5	Bayesian Inference for Several Parameters	151
3.8	Recommended Reading	151
4.	Putting the Math to Work: Linear Least-Squares Regression	152
4.1	Least-Squares Fit	152
4.2	A Statistical Model for Linear Regression	155
4.3	The Least-Squares Coefficients as Estimators	156
4.4	Statistical Inference for the Regression Model	157
4.5	Maximum-Likelihood Estimation of the Regression Model	160
4.6	Random Xs	161
References		165
Index		166