

# Contents

|   |          |
|---|----------|
| Introduction  | 1        |
| <b>Part 1. Classical theory of symmetric bilinear forms and quadratic forms</b> | <b>9</b> |
| Chapter I. Bilinear Forms   | 11       |
| 1. Foundations  | 11       |
| 2. The Witt and Witt-Grothendieck rings of symmetric bilinear forms             | 19       |
| 3. Chain equivalence  | 21       |
| 4. Structure of the Witt ring   | 22       |
| 5. The Stiefel-Whitney map  | 28       |
| 6. Bilinear Pfister forms   | 32       |
| Chapter II. Quadratic Forms   | 39       |
| 7. Foundations  | 39       |
| 8. Witt's Theorems  | 46       |
| 9. Quadratic Pfister forms I  | 52       |
| 10. Totally singular forms  | 55       |
| 11. The Clifford algebra  | 57       |
| 12. Binary quadratic forms and quadratic algebras                               | 60       |
| 13. The discriminant  | 61       |
| 14. The Clifford invariant  | 63       |
| 15. Chain $p$ -equivalence of quadratic Pfister forms                           | 64       |
| 16. Cohomological invariants  | 67       |
| Chapter III. Forms over Rational Function Fields                                | 71       |
| 17. The Cassels-Pfister Theorem   | 71       |
| 18. Values of forms   | 75       |
| 19. Forms over a discrete valuation ring  | 79       |
| 20. Similarities of forms   | 82       |
| 21. An exact sequence for $W(F(t))$   | 88       |
| Chapter IV. Function Fields of Quadrics   | 93       |
| 22. Quadrics  | 93       |
| 23. Quadratic Pfister forms II  | 98       |
| 24. Linkage of quadratic forms  | 101      |
| 25. The submodule $J_n(F)$  | 103      |
| 26. The Separation Theorem  | 107      |
| 27. A further characterization of quadratic Pfister forms                       | 109      |
| 28. Excellent quadratic forms   | 111      |

|   |            |
|---|------------|
| 29. Excellent field extensions                                      | 113        |
| 30. Central simple algebras over function fields of quadratic forms | 116        |
| Chapter V. Bilinear and Quadratic Forms and Algebraic Extensions    | 121        |
| 31. Structure of the Witt ring                                      | 121        |
| 32. Addendum on torsion   | 131        |
| 33. The total signature   | 133        |
| 34. Bilinear and quadratic forms under quadratic extensions         | 138        |
| 35. Torsion in $I^n(F)$ and torsion Pfister forms                   | 147        |
| Chapter VI. $u$ -invariants   | 161        |
| 36. The $\bar{u}$ -invariant  | 161        |
| 37. The $u$ -invariant for formally real fields                     | 165        |
| 38. Construction of fields with even $u$ -invariant                 | 170        |
| 39. Addendum: Linked fields and the Hasse number                    | 172        |
| Chapter VII. Applications of the Milnor Conjecture                  | 177        |
| 40. Exact sequences for quadratic extensions                        | 177        |
| 41. Annihilators of Pfister forms                                   | 181        |
| 42. Presentation of $I^n(F)$  | 184        |
| 43. Going down and torsion-freeness                                 | 188        |
| Chapter VIII. On the Norm Residue Homomorphism of Degree Two        | 193        |
| 44. The main theorem  | 193        |
| 45. Geometry of conic curves  | 194        |
| 46. Key exact sequence  | 198        |
| 47. Hilbert Theorem 90 for $K_2$                                    | 208        |
| 48. Proof of the main theorem                                       | 211        |
| <b>Part 2. Algebraic cycles</b>                                     | <b>215</b> |
| Chapter IX. Homology and Cohomology                                 | 217        |
| 49. The complex $C_*(X)$  | 217        |
| 50. External products   | 232        |
| 51. Deformation homomorphisms                                       | 235        |
| 52. $K$ -homology groups  | 238        |
| 53. Euler classes and projective bundle theorem                     | 243        |
| 54. Chern classes   | 247        |
| 55. Gysin and pull-back homomorphisms                               | 250        |
| 56. $K$ -cohomology ring of smooth schemes                          | 257        |
| Chapter X. Chow Groups  | 261        |
| 57. Definition of Chow groups                                       | 261        |
| 58. Segre and Chern classes   | 268        |
| Chapter XI. Steenrod Operations                                     | 277        |
| 59. Definition of the Steenrod operations                           | 278        |
| 60. Properties of the Steenrod operations                           | 281        |
| 61. Steenrod operations for smooth schemes                          | 283        |
| Chapter XII. Category of Chow Motives                               | 291        |

|   |            |
|---|------------|
| 62. Correspondences   | 291        |
| 63. Categories of correspondences                               | 295        |
| 64. Category of Chow motives                                    | 298        |
| 65. Duality   | 299        |
| 66. Motives of cellular schemes                                 | 300        |
| 67. Nilpotence Theorem  | 302        |
| <b>Part 3. Quadratic forms and algebraic cycles</b>             | <b>305</b> |
| Chapter XIII. Cycles on Powers of Quadrics                      | 307        |
| 68. Split quadrics  | 307        |
| 69. Isomorphisms of quadrics                                    | 309        |
| 70. Isotropic quadrics  | 310        |
| 71. The Chow group of dimension 0 cycles on quadrics            | 311        |
| 72. The reduced Chow group                                      | 313        |
| 73. Cycles on $X^2$   | 316        |
| Chapter XIV. The Izhboldin Dimension                            | 325        |
| 74. The first Witt index of subforms                            | 325        |
| 75. Correspondences   | 326        |
| 76. The main theorem  | 329        |
| 77. Addendum: The Pythagoras number                             | 332        |
| Chapter XV. Application of Steenrod Operations                  | 335        |
| 78. Computation of Steenrod operations                          | 335        |
| 79. Values of the first Witt index                              | 336        |
| 80. Rost correspondences  | 339        |
| 81. On the 2-adic order of higher Witt indices, I               | 342        |
| 82. Holes in $I^n$  | 347        |
| 83. On the 2-adic order of higher Witt indices, II              | 350        |
| 84. Minimal height  | 351        |
| Chapter XVI. The Variety of Maximal Totally Isotropic Subspaces | 355        |
| 85. The variety $\text{Gr}(\varphi)$                            | 355        |
| 86. The Chow ring of $\text{Gr}(\varphi)$ in the split case     | 356        |
| 87. The Chow ring of $\text{Gr}(\varphi)$ in the general case   | 361        |
| 88. The invariant $J(\varphi)$                                  | 364        |
| 89. Steenrod operations on $\text{Ch}(\text{Gr}(\varphi))$      | 366        |
| 90. Canonical dimension   | 367        |
| Chapter XVII. Motives of Quadrics                               | 371        |
| 91. Comparison of some discrete invariants of quadratic forms   | 371        |
| 92. The Nilpotence Theorem for quadrics                         | 373        |
| 93. Criterion of isomorphism                                    | 375        |
| 94. Indecomposable summands                                     | 378        |
| <b>Appendices</b>   | <b>381</b> |
| 95. Formally real fields  | 383        |
| 96. The space of orderings                                      | 384        |
| 97. $C_n$ -fields   | 385        |

|   |     |
|---|-----|
| 98. Algebras  | 387 |
| 99. Galois cohomology   | 393 |
| 100. Milnor $K$ -theory of fields                               | 397 |
| 101. The cohomology groups $H^{n,i}(F, \mathbb{Z}/m\mathbb{Z})$ | 402 |
| 102. Length and Herbrand index                                  | 407 |
| 103. Places   | 408 |
| 104. Cones and vector bundles                                   | 409 |
| 105. Group actions on algebraic schemes                         | 418 |
| Bibliography  | 421 |
| Notation  | 427 |
| Terminology   | 431 |