

Contents

1	Introduction	1
2	Preliminaries	5
2.1	Notation	5
2.2	Bayesian Nash Equilibrium	6
2.3	Auctions as Games	9
2.3.1	What is an Auction?	9
2.3.2	Auction Types	10
2.3.3	Auction as Bayesian Games	11
3	Private Values	13
3.1	The Independent Private Values Model	13
3.1.1	First-price Auctions	14
3.1.2	Second-price Auctions	18
3.1.3	Revenue Equivalence	20
3.1.4	Reserve Prices and Entry Fees	22
3.2	The Correlated Private Values Model	25
3.2.1	Second-price Auction	26
3.2.2	First-price Auction	27
3.2.3	Comparison of Expected Payment	29
3.3	The Effect of Risk Aversion	32
3.3.1	Revenue Comparison	33
3.4	The Discrete Valuation Case	34
3.5	Exercises	35
4	Common Value	39
4.1	An Example with Independent Signals	40
4.1.1	First-price Auction	40
4.1.2	Second-price Auction Example	42
4.2	An Example with Correlated Types	42
4.2.1	First-price Auction	43
4.2.2	Second-price Auction	45
4.3	The Symmetric Model with Two Bidders	46

4.3.1	Second-price Auctions	50
4.3.2	First-price Auctions	51
4.3.3	Revenue Comparison	54
4.4	Exercises	55
5	Affiliated Values	57
5.1	The General Model	58
5.2	Second-price Auctions	63
5.3	First-price Auctions	65
5.4	English Auctions	67
5.5	Expected Revenue Ranking	68
5.6	Exercises	70
6	Mechanism Design	71
6.1	The Revelation Principle	71
6.2	Direct Mechanisms	75
6.3	Revenue Equivalence and the Optimal Auction	79
6.4	Some Extensions	85
6.4.1	Non-monotonic Marginal Valuation	85
6.4.2	Correlated Values	97
6.4.3	Several Objects	111
6.4.4	Common Values Auction	113
6.5	Exercises	115
7	Multiple Objects	117
7.1	Sequential Auctions	118
7.2	Simultaneous Auctions	120
7.2.1	Discriminatory Auctions	121
7.2.2	Uniform price Auctions	126
7.3	Optimal Auction	133
7.4	Exercises	139
8	What is Next?	141
8.1	Distribution Hypotheses in Auction Theory	143
A	Probability	151
A.1	Probability Spaces	151
A.2	Uncountable Sample Space Case	153
A.3	Random Variables	154
A.4	Random Vectors and their Distribution	156
A.5	Independence of Random Variables	158
A.6	The Distribution of the Maximum of Independent Random Variables	158

<i>Contents</i>	xiii
A.7 The Distribution of the Second Highest Value	159
A.8 Mean Value of Random Variables	160
A.9 Conditional Probability	162
B Differential Equations	165
B.1 The Simplest Differential Equation	165
B.2 Integrating Factor	165
C Affiliation	167
D Convexity	171
References	175
Index of Notations	179
Index of Proper Names	181
Index	183