

Contents

1	Foundations of the Theory of Algebraic Function Fields	1
1.1	Places	1
1.2	The Rational Function Field	8
1.3	Independence of Valuations	12
1.4	Divisors	15
1.5	The Riemann-Roch Theorem	24
1.6	Some Consequences of the Riemann-Roch Theorem	31
1.7	Local Components of Weil Differentials	37
1.8	Exercises	40
2	Algebraic Geometry Codes	45
2.1	Codes	45
2.2	AG Codes	48
2.3	Rational AG Codes	55
2.4	Exercises	63
3	Extensions of Algebraic Function Fields	67
3.1	Algebraic Extensions of Function Fields	68
3.2	Subrings of Function Fields	77
3.3	Local Integral Bases	81
3.4	The Cotrace of Weil Differentials and the Hurwitz Genus Formula	90
3.5	The Different	99
3.6	Constant Field Extensions	112
3.7	Galois Extensions I	120
3.8	Galois Extensions II	130
3.9	Ramification and Splitting in the Compositum of Function Fields	137
3.10	Inseparable Extensions	143
3.11	Estimates for the Genus of a Function Field	145
3.12	Exercises	150

4 Differentials of Algebraic Function Fields	155
4.1 Derivations and Differentials	155
4.2 The P -adic Completion	161
4.3 Differentials and Weil Differentials	170
4.4 Exercises	179
5 Algebraic Function Fields over Finite Constant Fields	185
5.1 The Zeta Function of a Function Field	185
5.2 The Hasse-Weil Theorem	197
5.3 Improvements of the Hasse-Weil Bound	208
5.4 Exercises	212
6 Examples of Algebraic Function Fields	217
6.1 Elliptic Function Fields	217
6.2 Hyperelliptic Function Fields	224
6.3 Tame Cyclic Extensions of the Rational Function Field	227
6.4 Some Elementary Abelian p -Extensions of $K(x)$, char $K = p > 0$	232
6.5 Exercises	235
7 Asymptotic Bounds for the Number of Rational Places	243
7.1 Ihara's Constant $A(q)$	243
7.2 Towers of Function Fields	246
7.3 Some Tame Towers	259
7.4 Some Wild Towers	261
7.5 Exercises	285
8 More about Algebraic Geometry Codes	289
8.1 The Residue Representation of $C_\Omega(D, G)$	289
8.2 Automorphisms of AG Codes	290
8.3 Hermitian Codes	292
8.4 The Tsfasman-Vladut-Zink Theorem	297
8.5 Decoding AG Codes	303
8.6 Exercises	308
9 Subfield Subcodes and Trace Codes	311
9.1 On the Dimension of Subfield Subcodes and Trace Codes	311
9.2 Weights of Trace Codes	315
9.3 Exercises	325
Appendix A. Field Theory	327
Appendix B. Algebraic Curves and Function Fields	335
List of Notations	345

References	349
Index	353