Preface

Ich schaffe, was ihr wollt, und schaffe mehr;
Zwar ist es leicht, doch ist das Leichte schwer.
Es liegt schon da, doch um es zu erlangen,
Das ist die Kunst! Wer weiss es anzufangen?
GOETHE, Faust II

The present text centers around a fundamental task of measure and integration theory, which has not found an adequate solution so far. It is the task to produce, with unified and universal means, true contents and above all measures from more primitive data, in order to extend elementary contents and to represent so-called elementary integrals. The traditional main tools are the Carathéodory extension theorem and the Daniell-Stone representation theorem. These theorems are much too restrictive in order to fulfil the needs.

Around 1970 a new development started in the work of Topsøe and others. It was based on the notion of regularity, which for a set function means to determine its values from a particular set system by approximation from above or below. In traditional measure theory this notion is linked to topology.

The present text wants to be a systematic treatment of the context in the new spirit. It is based to some extent on personal work of the author. The main results are equivalence theorems for the existence and uniqueness of extensions and representations, which are not more complicated than the traditional ones but much more powerful. With these results the text clarifies and unifies the entire context. The main instruments are certain new envelope formations which resemble the traditional Carathéodory outer measure.

The systematic theory has numerous applications. The most important application is the full extension of the classical Riesz representation theorem in terms of Radon measures, from locally compact to arbitrary Hausdorff topological spaces. As another application we note an extension and at the same time simplification of the Choquet capacitability theorem, which shows that the new formations can be useful for so-called non-additive set functions as well. Some of the applications are treated without pronounced technical sophistication. We rather want to demonstrate that certain basic ideas and results are natural outflows from the new theory.

The central parts of the text are chapters II and V. Their main substance as well as their history and motivation are outlined in the introduction below. It is an elaboration of a lecture which the author delivered at several places, in the present form for the first time at the symposium in honour of Adriaan C.Zaanen in Leiden in September 1993.

Chapters I and IV are filled with preparations. We need certain standard material in unconventional versions which have to be developed. We also need several new notations.

The application to the Riesz representation theorem is in chapter V section 16. The other applications are in chapters III, VI and VII. We emphasize that chapter VII develops an abstract product formation which comprises the Radon product measure of Radon measures. The final chapter VIII is an appendix which is independent of the central chapters II and V. It wants to demonstrate that the unconventional notions of content and measure introduced in chapter I can be useful in other areas of measure theory as well.

All this says that the central themes of the present text are the fundamentals of measure and integration theory. The author hopes that its readers will find it less technical than it looks at first sight. He thinks that the text can be read with appreciation by anyone who has struggled through the traditional abstract and topological theories. However, it is different from a textbook in the usual sense. The presentation is ab ovo, though more like in a book of research. The author hopes that the text will be used in future courses. An ideal prerequisite would be the recent small book of Stroock [1994], because on the one hand it provides the concrete material which should precede this one, and on the other hand it does not take the reader onto the traditional paths of abstract measure and integration theory which the present work wants to restructure.

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