Preface to the Fourth Edition

This fourth edition of *Stochastic Methods* is thoroughly revised and augmented, and has been completely reset. While keeping to the spirit of the book I wrote originally, I have reorganised the chapters of Fokker-Planck equations and those on approximation methods, and introduced new material on the white noise limit of driven stochastic systems, and on applications and validity of simulation methods based on the Poisson representation. Further, in response to the revolution in financial markets following from the discovery by Fischer Black and Myron Scholes of a reliable option pricing formula, I have written a chapter on the application of stochastic methods to financial markets. In doing this, I have not restricted myself to the geometric Brownian motion model, but have also attempted to give some flavour of the kinds of methods used to take account of the realities of financial markets. This means that I have also given a treatment of Lévy processes and their applications to finance, since these are central to most current thinking.

Since this book was written the rigorous mathematical formulation of stochastic processes has developed considerably, most particularly towards greater precision and generality, and this has been reflected in the way the subject is presented in modern applications, particularly in finance. Nevertheless, I have decided to adhere to my original decision, to use relatively simple language without excessive rigour; indeed I am not convinced that the increase in rigour and precision has been of significant help to those who want to use stochastic methods as a practical tool.

The new organisation of the material in the book is as in the figure on the next page. Instead of the original ten chapters, there are now fifteen. Some of the increase is a result of my decision to divide up some of the larger chapters into tighter and more logically structured smaller chapters, but Chapters 8 and 10 are completely new. The basic structure of the book is much the same, building on the basis of Ito stochastic differential equations, and then extending into Fokker-Planck equations and jump processes. I have put all of the work on the Poisson representation into a single chapter, and augmented this chapter with new material.

Stochastic Methods, although originally conceived as a book for physicists, chemists and similar scientists, has developed a readership with far more varied tastes, and this new edition is designed to cater better for the wider readership, as well as to those I originally had in mind. At the same time, I have tried hard to maintain "look and feel" of the original, and the same degree of accessibility.

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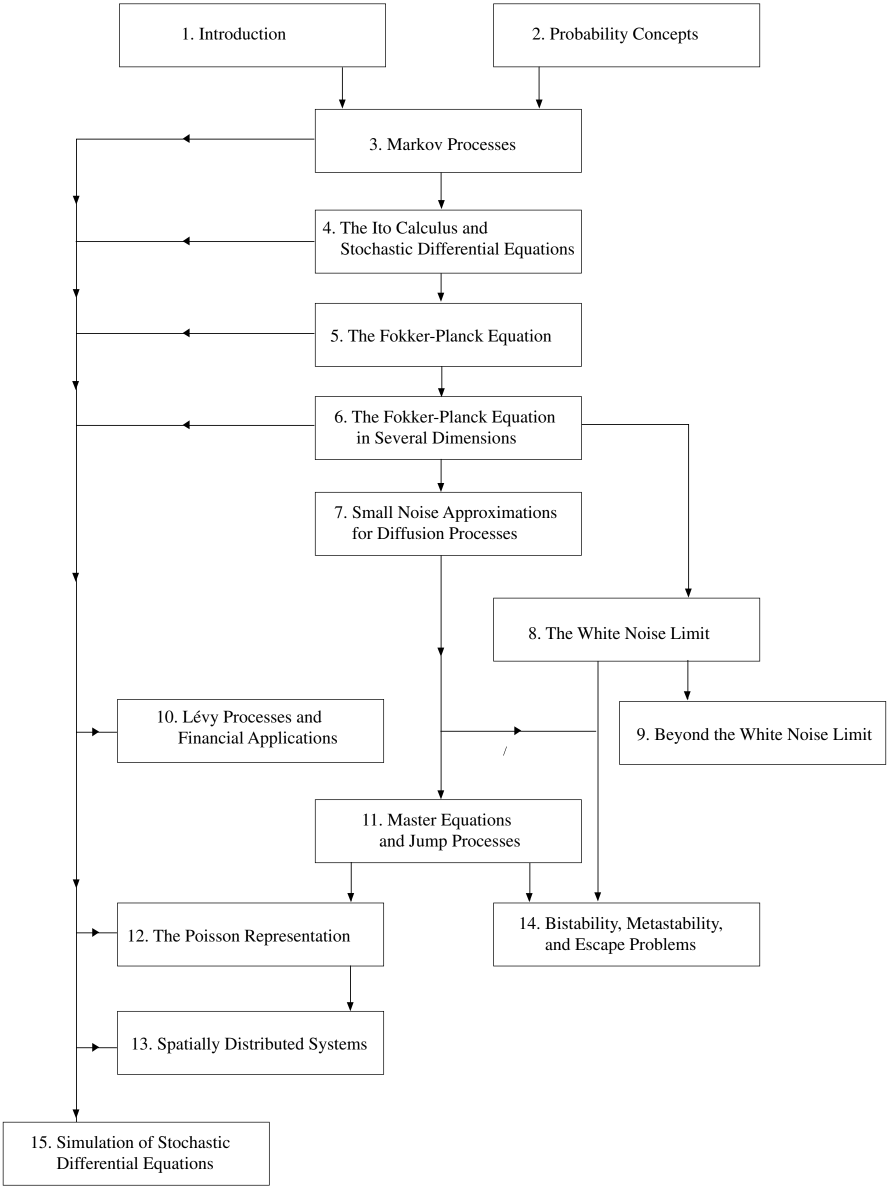
From the Preface to the First Edition

My intention in writing this book was to put down in relatively simple language and in a reasonably deductive form, all those formulae and methods which have been


1. Introduction
2. Probability Concepts
3. Markov Processes
4. The Ito Calculus and Stochastic Differential Equations
5. The Fokker-Planck Equation
6. The Fokker-Planck Equation in Several Dimensions
7. Small Noise Approximations for Diffusion Processes
8. The White Noise Limit
9. Beyond the White Noise Limit
10. Lévy Processes and Financial Applications
11. Master Equations and Jump Processes
12. The Poisson Representation
13. Spatially Distributed Systems
14. Bistability, Metastability, and Escape Problems
15. Simulation of Stochastic Differential Equations

scattered throughout the scientific literature on stochastic methods throughout the eighty years that they have been in use. This might seem an unnecessary aim since there are scores of books entitled "Stochastic Processes", and similar titles, but careful perusal of these soon shows that their aim does not coincide with mine. There are purely theoretical and highly mathematical books, there are books related to electrical engineering or communication theory, and there are books for biologists—many of them very good, but none of them covering the kind of applications that appear nowadays so frequently in Statistical Physics, Physical Chemistry, Quantum Optics and Electronics, and a host of other theoretical subjects.

The main new point of view here is the amount of space which deals with methods of approximating problems, or transforming them for the purpose of approximating them. I am fully aware that many workers will not see their methods here. But my criterion here has been whether an approximation is *systematic*. Many approximations are based on unjustifiable or uncontrollable assumptions, and are justified *a posteriori*. Such approximations are not the subject of a systematic book—at least, not until they are properly formulated, and their range of validity controlled. In some cases I have been able to put certain approximations on a systematic basis, and they appear here—in other cases I have not.

A word on the background assumed. The reader must have a good knowledge of practical calculus including contour integration, matrix algebra, differential equations, both ordinary and partial, at the level expected of a first degree in applied mathematics, physics or theoretical chemistry.

I expect the readership to consist mainly of theoretical physicists and chemists, and thus the general standard is that of these people. This is not a rigorous book in the mathematical sense, but it contains results, all of which I am confident are provable rigorously, and whose proofs can be developed out of the demonstrations given. The organisation of the book is as in the following table, and might raise some eyebrows. For, after introducing the general properties of Markov processes, I have chosen to base the treatment on the conceptually difficult but intuitively appealing concept of the stochastic differential equation. I do this because of my own experience of the simplicity of stochastic differential equation methods, once one has become familiar with the Ito calculus, which I have presented in Chap. 4 in a rather straightforward manner, such as I have not seen in any previous text.

For the sake of compactness and simplicity I have normally presented only one way of formulating certain methods. For example, there are several different ways of formulating the adiabatic elimination results, though few have been used in this context. To have given a survey of all formulations would have required an enormous and almost unreadable book. However, where appropriate I have included specific references, and further relevant matter can be found in the general bibliography.

Hamilton, New Zealand *C.W. Gardiner*
January, 1983