Contents

Chapters marked with * may form the matter of a basic introductory course

Part I Stellar Equilibrium With and Without Rotation

1 The Mechanical Equilibrium of Stars* 3
1.1 Momentum and Continuity Equations 3
1.1.1 Hydrodynamical Equations 3
1.1.2 Hydrostatic Equilibrium 5
1.1.3 Mass Conservation and Continuity Equation 5
1.1.4 Lagrangian and Eulerian Variables 6
1.1.5 Estimates of Pressure, Temperature and Timescales 7
1.2 The Potential Energy 10
1.2.1 Relation to the Potential and Poisson Equation 11
1.2.2 The Potential Energy as a Function of Pressure 12
1.2.3 The Internal Stellar Temperature 13
1.3 The Virial Theorem for Stars 13
1.3.1 Star with Perfect Gas Law 14
1.3.2 Star with a General Equation of State 16
1.3.3 Slow Contraction, the Kelvin–Helmholtz Timescale 17

2 The Mechanical Equilibrium of Rotating Stars 19
2.1 Equilibrium Configurations 19
2.1.1 From Maclaurin Spheroids to the Roche Models 19
2.1.2 Hydrostatic Equilibrium for Solid Body Rotation 20
2.1.3 Stellar Surface and Gravity 22
2.1.4 Critical Velocities 24
2.1.5 Polar Radius as a Function of Rotation 27
2.2 Equations of Stellar Structure for Shellular Rotation 29
2.2.1 Properties of the Isobars 30
2.2.2 Hydrostatic Equilibrium 31
2.2.3 Continuity Equation 32
2.2.4 Equation of the Surface for Shellular Rotation 33

3 The Energetic Equilibrium of Stars* 35
3.1 The Radiative Transfer 35
3.1.1 Equation of Radiative Transfer 36
3.1.2 Radiation Properties in Stellar Interiors 37
3.1.3 Transfer Equation 38
3.1.4 The Rosseland Mean Opacity 40
3.1.5 The Mass–Luminosity Relation 41
3.1.6 Photon Travel Times and *M–L* Relation 42
3.2 Energetic Equilibrium of a Star 43
3.2.1 Why Are Stars Stable Nuclear Reactors? 43
3.2.2 Energy Conservation 44
3.2.3 Combined Equation of Conservation and Transfer 45
3.2.4 Relation with the Heat Conduction 46
3.3 Energy Generation Rate from Gravitational Contraction. Thermodynamic Expressions of dq 47
3.3.1 Contraction of a Star with Perfect Gas 48
3.3.2 Case of a General Equation of State 48
3.3.3 The Entropy of Mixing 50
3.3.4 The Difference of Specific Heats 51
3.3.5 Adiabatic Gradient for Constant μ 52
3.3.6 Adiabatic Gradient for Variable μ 53
3.4 Changes of T and ϱ for Non-adiabatic Contraction 53
3.4.1 Major Consequences for Evolution 54
3.5 Secular Stability of Nuclear Burning 56
3.5.1 Shell Source Instability 57
3.6 The Role of Radiation Pressure in Stars 58
3.6.1 The Radiative Pressure as a Function of Mass 60
3.6.2 The Eddington Luminosity 63

4 The Energy Conservation and Radiative Equilibrium in Rotating Stars 67
4.1 Radiative Equilibrium for Rotating Stars 67
4.1.1 The Equation of Radiative Transfer 67
4.1.2 Conservation of Energy 68
4.1.3 Structure Equations for Rotating Stars 68
4.2 Radiative Transfer in Rotating Stars 70
4.2.1 Breakdown of Radiative Equilibrium 70
4.2.2 The Von Zeipel Theorem 71
4.2.3 Interferometric Observations of Stellar Distortion and Gravity Darkening 74
4.3 Interactions of Rotation and Radiation Effects 75
4.3.1 The Γ, Ω and $\Omega\Gamma$ Limits 75
4.3.2 The $\Omega\Gamma$ Limit: Combined Eddington and Rotation Limits 76
4.4 Critical Rotation Velocities 78
4.4.1 No Break-up Velocity for Differential Rotation ? 78

4.4.2 Classical Expression of the Critical Velocity 79
4.4.3 The Different Rotation Parameters 80
4.4.4 Critical Velocity Near the Eddington Limit 81

5 Stellar Convection* 83
5.1 Gravity Waves and the Brunt–Väisälä Frequency 83
5.1.1 Relation with the Entropy Gradient 85
5.1.2 The Schwarzschild and Ledoux Criteria 86
5.1.3 The Four T Gradients 88
5.2 Mixing-Length Theory for the Convective Flux 90
5.2.1 Orders of Magnitude 92
5.3 Convection in Stellar Interiors 94
5.4 Non-adiabatic Convection 97
5.4.1 Radiative Losses 97
5.4.2 Thermal Adjustment Timescale 98
5.4.3 Solutions for Non-adiabatic Convection 99
5.4.4 Limiting Cases, Fraction Carried by Convection 100
5.5 Convection in the Most Luminous Stars 103
5.5.1 Convection Near the Eddington Limit 103
5.5.2 Density Inversion 104
5.5.3 Pressure and Flux of Turbulence 104

6 Overshoot, Semiconvection, Thermohaline Convection, Rotation and Solberg–Hoiland Criterion 109
6.1 Convective Overshooting 109
6.1.1 Overshooting in an MLT Non-local Model 109
6.1.2 The Roxburgh Criterion for Convective Overshoot 112
6.1.3 Turbulence Modeling and Overshooting 114
6.1.4 Observational Constraints 116
6.2 Semiconvection and Thermohaline Convection 118
6.2.1 Various Approaches 119
6.2.2 Kato Equation, Thermohaline Convection 121
6.2.3 Diffusion Coefficient for Semiconvection 123
6.3 Time-Dependent Convection 124
6.4 Effects of Rotation on Convection 125
6.4.1 Oscillation Frequency in a Rotating Medium 125
6.4.2 The Rayleigh Criterion and Rayleigh–Taylor Instability 127
6.4.3 The Solberg–Hoiland Criterion 128
6.4.4 Numerical Simulations 129
6.5 Convective Envelope in Rotating O-stars 130

Part II Physical Properties of Stellar Matter

7 The Equation of State* 137
7.1 Excitation and Ionization of Gases 137
7.1.1 Excitation 137
7.1.2 Ionization of Gases: The Saha Equation 139
7.1.3 The Saha–Boltzmann Equation 140
7.1.4 Ionization Potentials and Negative Ions 142
7.2 Perfect Gas and Mean Molecular Weights 143
7.3 Partially Ionized Stellar Medium 145
7.3.1 Coupled Equations for a Medium Partially Ionized 146
7.3.2 Thermodynamic Coefficients for Partial Ionization 148
7.4 Adiabatic Exponents and Thermodynamic Functions 149
7.4.1 Definitions of the Adiabatic Exponents 149
7.4.2 Relation Between the Γ_i and Specific Heats 151
7.5 Thermodynamics of Mixture of Gas and Radiation 153
7.6 Electrostatic Effects 156
7.6.1 The Debye–Hückel Radius 156
7.6.2 Electrostatic Effects on the Gas Pressure 158
7.6.3 Ionization by Pressure 159
7.6.4 Crystallization 160
7.7 Degenerate Gases 161
7.7.1 Partially Degenerate Gas 164
7.7.2 Non-Relativistic Partial Degeneracy 164
7.7.3 Completely Degenerate Gas 166
7.7.4 Electrostatic Effects in a Degenerate Medium 169
7.7.5 A Note on the Consequences of Degeneracy and on White Dwarfs 170
7.8 Global View on the Equation of State 172

8 The Opacities* 177
8.1 Line Absorption, Electron Scattering, Rayleigh Diffusion 177
8.1.1 Recalls on the Atomic Oscillators 177
8.1.2 Spectral Lines or Bound–Bound Transitions 178
8.2 Electron Scattering 179
8.2.1 Electron Scattering at High Energies 180
8.2.2 Rayleigh Diffusion 181
8.3 Photoionization or Bound–Free Transitions 181
8.3.1 Negative H Absorption 183
8.4 Hyperbolic Transitions or Free–Free Opacity 185
8.5 Electronic Conduction 186
8.5.1 Electron Conduction in Non-degenerate Gas 186
8.5.2 Electron Conduction in Degenerate Gas 188
8.6 Global View on Stellar Opacities 189
8.6.1 Dependence on T and ϱ, Changes with Masses 189
8.6.2 Opacity Tables 191

9 Nuclear Reactions and Neutrino Processes* 193
9.1 Physics of the Nuclear Reactions 193
9.1.1 Reaction Energy 193
9.2 Nuclear Reaction Rates 195
9.2.1 Particle Lifetimes and Energy Production Rates 196
9.3 Nuclear Cross-Sections 196
9.3.1 The Rate of Non-resonant Reactions 200
9.3.2 The Rate of Resonant Nuclear Reactions 204
9.4 Electron Screening 205
9.5 Neutrino Emission Processes 207
9.5.1 Photo-neutrinos 208
9.5.2 Pair Annihilation Neutrinos 208
9.5.3 Plasma, Bremsstrahlung, Recombination Neutrinos 211

Part III Hydrodynamical Instabilities and Transport Processes

10 Transport Processes: Diffusion and Advection 215
10.1 General Properties of Diffusion 216
10.1.1 Absence of Global Mass Flux 216
10.1.2 Continuity Equation: Atomic Diffusion and Motion 217
10.1.3 Fluxes of Particles, Velocities and Diffusion Coefficient 218
10.2 Diffusion by an Abundance Gradient 221
10.2.1 Equation of Diffusion 221
10.2.2 Boundary Conditions and Interpolations 223
10.2.3 Caution About the Use of Concentrations 224
10.3 Microscopic or Atomic Diffusion 225
10.3.1 Gravitational Settling 225
10.3.2 Equations of Motion of Charged Particles 227
10.3.3 The Electric Field and the Diffusion Velocities 229
10.3.4 Diffusion Equation 231
10.3.5 Effect of a Thermal Gradient 232
10.4 The Radiative Diffusion 233
10.4.1 Radiative Acceleration 233
10.4.2 Acceleration by Spectral Lines 235
10.4.3 Continuum Absorption, Redistribution, Magnetic Field 236
10.4.4 Orders of Magnitude, Diffusion in A Stars 237
10.4.5 Atomic Diffusion in the Sun 238
10.5 Transport of Angular Momentum in Stars 239
10.5.1 Equation of Transport 239
10.5.2 Transport of Angular Momentum by Shears 241
10.5.3 Some Properties of Shellular Rotation 242
10.5.4 Transport in Shellular Rotation 244
10.5.5 Boundary Conditions 246

11 Meridional Circulation 249
11.1 The Energy Conservation on an Isobar 249
11.1.1 Thermal Imbalance 250
11.1.2 The Horizontal Thermal Balance 253
11.2 Some Properties of Baroclinic Stars 255
11.2.1 The Fluctuations of T, μ, ε and χ 255
11.2.2 The Baroclinic Equation 258
11.2.3 The Horizontal Fluctuations of Effective Gravity 259
11.3 The Velocity of Meridional Circulation 262
11.4 Properties of Meridional Circulation 267
11.4.1 Simplified Expressions and Timescale 267
11.4.2 T and μ Excesses and Circulation Patterns 269
11.5 The Major Role of the Gratton–Öpik Term 272
11.5.1 Departure from Solid Body and Initial Ω Convergence 272
11.5.2 Stationary Circulation in Equilibrium with Diffusion 273
11.5.3 The Gratton–Öpik Circulation and Evolution 274
11.6 Meridional Circulation with Horizontal Turbulence 276
11.6.1 Transport of the Elements 277
11.6.2 Transport of the Angular Momentum 281

12 Rotation-Driven Instabilities 283
12.1 Horizontal Turbulence 283
12.1.1 The Horizontal Fluctuations of Ω 284
12.1.2 A First Estimate of the Horizontal Turbulence 287
12.1.3 Turbulent Diffusion from Laboratory Experiment 288
12.1.4 What Sets the Timescale of Horizontal Turbulence ? 290
12.1.5 Consequences 291
12.2 Shear Instabilities and Mixing 293
12.2.1 The Richardson Criterion 294
12.2.2 Dynamical Shears 296
12.2.3 Thermal Effects at Constant μ 296
12.2.4 The T Gradient in Shears 299
12.2.5 Thermal Effects and μ Gradient 300
12.3 Shear Mixing with Horizontal Turbulence 302
12.3.1 Richardson Criterion with Horizontal Turbulence 302
12.3.2 The Coefficient of Shear Diffusion with Turbulence 303
12.4 Baroclinic Instabilities 305
12.4.1 The Goldreich–Schubert–Fricke or GSF Instability 306
12.4.2 The ABCD Instability 308

13 Magnetic Field Instabilities and Transport Processes 311
13.1 The Equations of Magnetohydrodynamics (MHD) 311
13.1.1 The MHD Equations in Astrophysics 311
13.1.2 Equations of Stellar Structure with Magnetic Field 313
13.1.3 Alfvén Waves 315

13.1.4 Dynamos and the Solar Dynamo 316
13.1.5 Observed Fields and Limits 317
13.2 Magnetic Braking of Rotating Stars 319
13.2.1 Saturation Effects 321
13.2.2 Mass Dependence 322
13.2.3 Consequences 323
13.3 Magnetic Field Properties in Radiative Regions 325
13.3.1 The Ferraro Law of Isorotation 326
13.3.2 Field Amplification by Winding-Up 327
13.3.3 Magnetic Field Evolution and Rotational Smoothing 328
13.4 The Tayler Instability and Possible Dynamo 330
13.4.1 The Tayler Instability 330
13.4.2 The Tayler–Spruit Dynamo and Questions 331
13.4.3 Conditions for Instability 332
13.4.4 Thermal Diffusivity 334
13.4.5 Solutions of the Dynamo Equations 336
13.5 Transports of Angular Momentum by the Magnetic Field 339
13.5.1 Viscous Coupling by the Field 339
13.5.2 Horizontal Coupling of Rotation 340
13.5.3 Check for Consistency 341
13.6 Models with Magnetic Field and Circulation 342
13.6.1 Evolution of Ω, B and the Diffusion Coefficients 343
13.6.2 Evolutionary Consequences 347
13.7 Other Magnetic Instabilities 348
13.7.1 Magnetic Shear Instability and Transport 349
13.7.2 Magnetic Buoyancy 351

14 Physics of Mass Loss by Stellar Winds 355
14.1 Stellar Wind Properties 355
14.2 Radiatively Line-Driven Winds 357
14.2.1 Simplified Theory 357
14.2.2 Metallicity, Velocities and Other Effects 361
14.3 Kudritzki's Wind Momentum–Luminosity Relation 362
14.3.1 Rotation and the WLR Relation 363
14.4 Rotation Effects on Stellar Winds 364
14.4.1 Latitudinal Variations 364
14.4.2 Mass Loss and Rotation 367

Part IV Acoustic and Gravity Waves. Helio- and Asteroseismology

15 Radial Pulsations of Stars 371
15.1 Thermodynamics of the Pulsations 371
15.2 Linear Analysis of Radial Oscillations 373
15.2.1 Convection 378
15.2.2 Boundary Conditions and Eigenvalue Problem 379

15.3 Baker's One-Zone Analytical Model . . . 381
15.3.1 Adiabatic Pulsations . . . 384
15.4 Non-adiabatic Effects in Pulsations . . . 385
15.4.1 The κ and γ Mechanisms . . . 385
15.4.2 The Damping Timescale of Pulsations . . . 389
15.4.3 Secular Instability: Conditions on Opacities and Nuclear Reactions . . . 390
15.5 Relations to Observations: Cepheids . . . 391
15.5.1 The Period-Luminosity-Color Relations . . . 391
15.5.2 Physics of the Instability Strip . . . 394
15.5.3 The Period–Luminosity Relation . . . 397
15.5.4 Light Curves . . . 398

16 Nonradial Stellar Oscillations . . . 401
16.1 Basic Equations of Nonradial Oscillations . . . 401
16.1.1 Starting Equations . . . 401
16.1.2 Perturbations of the Equations . . . 402
16.1.3 Separation in Vertical and Horizontal Components . . . 404
16.1.4 Decomposition in Spherical Harmonics . . . 405
16.2 Nonradial Adiabatic Oscillations . . . 409
16.2.1 Basic Equations . . . 409
16.2.2 Some Properties of the Equations . . . 411
16.2.3 Simplification to a Second-Order Equation . . . 412
16.2.4 Domains of the Acoustic and Gravity Modes . . . 414
16.2.5 The Degree ℓ and Radial Order n . . . 416
16.3 Properties of Acoustic or p Modes . . . 419
16.3.1 Inner Turning Points of p Modes . . . 419
16.3.2 Properties of the Solar Cavity: Parabolic Relations . . . 421
16.3.3 Behavior of p Modes at the Surface . . . 423
16.3.4 Excitation and Damping . . . 427
16.4 The Asymptotic Theory of p Modes . . . 428
16.4.1 The Frequencies of p Modes . . . 428
16.4.2 Second-Order Effects . . . 431
16.5 Helioseismology and Asteroseismology . . . 433
16.5.1 Helioseismic Observations . . . 433
16.5.2 Asteroseismic Observations . . . 436
16.5.3 The Asteroseismic Diagram . . . 438
16.5.4 Effects of X, Z and Mixing Length on the Large and Small Separations . . . 441
16.6 Rotational Effects: Splitting and Internal Mixing . . . 441
16.6.1 The Rotational Splitting: First Approach . . . 442
16.6.2 Further Steps . . . 443
16.6.3 The Tachocline and Inner Solar Rotation . . . 444
16.6.4 Structural Effects of Rotation . . . 447

17 Transport by Gravity Waves 449
17.1 The Propagation of Gravity Waves 449
17.1.1 Properties of Gravity Waves 449
17.1.2 Propagation Equation 451
17.1.3 Non-adiabatic Effects 454
17.2 Energy and Momentum Transport by Gravity Waves 458
17.2.1 Wave Excitation 461
17.3 Consequences of Transport by Gravity Waves 465
17.3.1 Shear Layer Oscillations "SLO" 465
17.3.2 The Solar Rotation Curve 467
17.3.3 Waves and the Lithium Dip 469
17.4 Transport by Gravity Waves and Open Questions 470
17.4.1 Particles Diffusion by Gravity Waves 470
17.4.2 Open Questions and Further Developments 471

Part V Star Formation

18 Pre-stellar Phase* 475
18.1 Overview and Signatures of Star Formation 475
18.2 The Beginning of Cloud Contraction 477
18.2.1 The Jeans Criterion 477
18.2.2 Various Expressions of the Jeans Criterion 480
18.2.3 Initializing the Cloud Collapse 482
18.2.4 The Timescale 482
18.3 The Role of Magnetic Field and Turbulence 484
18.3.1 Magnetic Fields 484
18.3.2 The Major Role of Turbulence in Star Formation 486
18.4 Isothermal Collapse and Cloud Fragmentation 487
18.4.1 Dust Grains and Cooling 487
18.4.2 The Initial Cloud Structure and its Evolution 488
18.4.3 The Hierarchical Fragmentation 491
18.4.4 The Opacity-Limited Fragmentation 492
18.4.5 The Initial Stellar Mass Spectrum 494

19 The Protostellar Phase and Accretion Disks* 497
19.1 Accretion Disks 497
19.1.1 Observations of Disks 497
19.1.2 Disk Formation 498
19.1.3 Disk Properties and Evolution 500
19.1.4 Stationary Disks 502
19.2 Accretion in Low and Intermediate Mass Stars 504
19.2.1 Theoretical Estimates of the Accretion Rates 505
19.2.2 Structure of the Protostar in the Accretion Phase 506
19.3 The Phase of Adiabatic Contraction 508
19.3.1 Evolution of the Central Object 510
19.4 Properties at the End of the Protostellar Phase 511

20 The Pre-main Sequence Phase and the Birthlines* 513
20.1 General Properties of Non-adiabatic Contraction 513
20.1.1 The Kelvin–Helmholtz Timescale 513
20.2 Pre-MS Evolution at Constant Mass 514
20.2.1 The Hayashi Line 514
20.2.2 Gravitational Energy Production and D Burning 515
20.2.3 From the Hayashi Line to the ZAMS 519
20.3 Pre-MS Evolution with Mass Accretion 520
20.3.1 The Birthline and Its Timescales 520
20.3.2 The Luminosity from D Burning 521
20.4 Evolution on the Birthline 521
20.5 Evolution from the Birthline to the ZAMS 526
20.6 Lifetimes, Ages and Isochrones 529
20.7 Lithium Depletion in Pre-MS Stars 531
20.7.1 Model Predictions 531
20.7.2 Li and D in T Tauri Stars and Residual Accretion 533
20.7.3 Li Depletion in Low-Mass Stars and Brown Dwarfs 534
20.7.4 Li Dating from Brown Dwarfs and Low-*M* Stars 536

21 Rotation in Star Formation 539
21.1 Steps in the Loss of Angular Momentum 539
21.1.1 From Interstellar Clouds to T Tauri Stars 540
21.1.2 From T Tauri Stars to the ZAMS 541
21.1.3 End of Pre-MS Phase and Early Main Sequence 542
21.2 Disk Locking and Magnetospheric Accretion 543
21.2.1 Observational Evidences 545
21.3 Magnetic Braking and Rotation in Clusters 545
21.3.1 Predicted Magnetic Braking 545
21.3.2 Comparisons with Rotation Velocities in Clusters 547

22 The Formation of Massive Stars 551
22.1 The Various Scenarios for Massive Star Formation 551
22.1.1 The Classical or Constant Mass Scenario 551
22.1.2 The Collision or Coalescence Scenario 552
22.1.3 The Accretion Scenario 554
22.2 Timescales for Accreting Stars 557
22.3 Limits on the Accretion Rates 558
22.3.1 The Upper Limit on Accretion 558
22.3.2 Conditions on Dust Opacity 561
22.3.3 The Lower Limit on Accretion Rates 562
22.3.4 The Role of Rotation 563
22.4 Accretion Models for Massive Star Formation 565
22.4.1 Formation with Initially Peaked Accretion 565
22.4.2 The Churchwell–Henning Relation 570

23 The Formation of First Stars in the Universe: Pop. III and Pop. II.5 Stars 571
23.1 The Pre- and Protostellar Phases at $Z = 0$ 572
23.1.1 Molecular H_2 and Gas Cooling 572
23.1.2 Fragmentation of Metal-Free Clouds 573
23.1.3 Formation of an Adiabatic Core 574
23.1.4 Accretion on the Core 575
23.2 The Mass–Radius Relation of $Z = 0$ Stars 576
23.3 Evolution of the Largest Masses at $Z = 0$ 579
23.3.1 Critical Accretion for Massive Stars at $Z = 0$ 580
23.4 The HR Diagram of Accreting Stars at $Z = 0$ 581
23.4.1 The Case of Non-zero Metallicities 581
23.4.2 The Role of Rotation 582
23.5 The Upper Mass Limit at $Z = 0$ 582
23.5.1 Main Effects 583
23.5.2 HII Region in a Free-Falling Envelope 583
23.5.3 Radiation Effect on an HII Region 585
23.6 The Pop. II.5 Stars 587
23.6.1 HD Formation and Gas Cooling 587
23.6.2 The Masses of the Pop. II.5 Stars 588

Part VI Main-Sequence and Post-MS Evolution

24 Solutions of the Equations and Simple Models* 593
24.1 Hydrostatic and Hydrodynamic Models 593
24.1.1 Hydrostatic Models and Vogt–Russel Theorem 593
24.1.2 Hydrodynamic Equations 595
24.1.3 Boundary Conditions at the Center and Surface 596
24.1.4 Analytical Solutions in the Outer Layers 597
24.2 The Henyey Method 599
24.3 Homology Transformations: Relations *M–L–R* 601
24.3.1 Other Effects: Electron Scattering, $P_{\rm rad}$, Convection 604
24.4 The Helium and Generalized Main Sequences 605
24.4.1 The Helium Sequence 605
24.4.2 Generalized Main Sequences 605
24.5 Polytropic Models 607
24.5.1 Interesting Polytropes 609
24.5.2 Isothermal Sphere 610

25 Evolution in the H-Burning Phases* 613
25.1 Hydrogen Burning 613
25.1.1 The pp Chains 614
25.1.2 Equations for Composition Changes 615
25.1.3 The CNO Cycles 618
25.1.4 Energy Production in MS Stars 620
25.1.5 The NeNa and MgAl Cycles 621

25.2 Basic Properties of MS Stars 624
25.2.1 Differences in Structure 624
25.2.2 Main Parameters as a Function of Mass 625
25.2.3 Evolutionary Timescales 627
25.3 Solar Properties and Evolution 629
25.3.1 Internal Structure 629
25.3.2 The Evolution of the Sun 632
25.3.3 Solar Neutrinos 634
25.4 Evolution on the Main Sequence 637
25.4.1 Internal Properties, Tracks in the HR Diagram 637
25.5 The End of the Main Sequence 640
25.5.1 The Schönberg–Chandrasekhar Limit 640
25.5.2 Isochrones and Age Determinations 642

26 Evolution in the He Burning and AGB Phases of Low and Intermediate Mass Stars with Rotation* 645
26.1 Helium Burning 645
26.2 He Burning in Intermediate Mass Stars 647
26.2.1 From Main Sequence to Red Giants 647
26.2.2 Evolution in the He-Burning Phase and Dredge-up 651
26.2.3 From AGB to the White Dwarfs 654
26.2.4 The Blue Loops 656
26.3 Some Metallicity Effects in Evolution 657
26.4 Central Evolution and Domains of Stellar Masses 658
26.4.1 The Mass Limits for Evolution 661
26.4.2 Evolution of the Entropy per Baryon 664
26.5 The Horizontal Branch 665
26.6 Evolution and Nucleosynthesis in AGB Stars 667
26.6.1 Structure and Instability of TP-AGB Stars 667
26.6.2 Third Dredge-Up and TP-AGB Nucleosynthesis 671
26.6.3 AGB Classification and Chemical Abundances 674
26.6.4 Post-AGB Stars to Planetary Nebulae and White Dwarfs, Super-AGB Stars 676
26.7 Rotation and Mixing Effects in AGB stars 678
26.8 Nucleosynthesis in AGB Stars 681
26.8.1 Nucleosynthesis in E-AGB stars 681
26.8.2 Nucleosynthesis in TP-AGB Stars 683

27 Massive Star Evolution with Mass Loss and Rotation* 685
27.1 The Need for Both 685
27.2 Evolution at Constant Mass 686
27.3 Internal Evolution and the HR Diagram 688
27.3.1 Mass Loss Parametrizations 688
27.3.2 Mass Loss Effects in the HR Diagram 690
27.3.3 Internal Evolution with Mass Loss 692

27.3.4 Effects of Rotation in the MS Phase . . . 693
27.3.5 Lifetimes and Age Estimates . . . 694
27.3.6 He-Burning: Blue and Red Supergiants at Different Z . . . 696
27.4 Evolution of the Chemical Abundances . . . 697
27.4.1 Steps in the Peeling-Off by Mass Loss . . . 697
27.4.2 Observed N/H Excesses . . . 699
27.4.3 Chemistry in Models with Rotation . . . 700
27.4.4 Abundances and Massive Star Filiations . . . 702
27.5 Wolf–Rayet Stars: the Daughters of O stars . . . 703
27.5.1 WR Properties: the Zebras in the Zoo . . . 703
27.5.2 Optically Thick Winds. M–L–R–T_{eff} Relations . . . 703
27.6 WR Star Chemistry . . . 706
27.6.1 Observations . . . 706
27.6.2 Mass Loss, Rotation and WR Chemistry . . . 707
27.6.3 ^{22}Ne in WC Stars . . . 709
27.7 Number Ratios of WR Stars in Galaxies . . . 710
27.7.1 Observed Number Ratios . . . 710
27.7.2 Models with Mass Loss and Rotation . . . 711
27.8 Evolution of the Rotational Velocities . . . 713
27.8.1 Rotation of LBV . . . 715
27.8.2 WR Star Rotation . . . 716

28 Advanced Evolutionary Stages and Pre-supernovae* . . . 719
28.1 Nuclear Reactions in the Advanced Phases . . . 719
28.1.1 C Burning . . . 719
28.1.2 Ne Photodisintegration . . . 723
28.1.3 O Burning . . . 723
28.1.4 Silicon Burning . . . 724
28.2 The Advanced Phases with and Without Rotation . . . 725
28.2.1 Toward the "Onion Skin" Model . . . 726
28.2.2 Decoupling of Core and Envelope . . . 727
28.2.3 Evolution of Central Conditions . . . 727
28.2.4 Lifetimes and Core Masses, Rotation . . . 729
28.3 Chemical Yields: Z, Mass Loss and Rotation Effects . . . 731
28.3.1 Chemical Yields of α-Rich Nuclei . . . 732
28.4 Toward the Supernovae . . . 734
28.4.1 Supernova Types . . . 734
28.4.2 Core Collapse and Explosion . . . 736
28.4.3 Final Masses and Remnants . . . 738
28.5 Explosive Synthesis . . . 741
28.5.1 Elements with A $<$ 56 . . . 741
28.5.2 The Fe Peak . . . 743
28.5.3 The Heavy Elements A $\geq$ 60 . . . 745
28.5.4 The s-Elements . . . 746
28.5.5 The r-Elements . . . 747

28.6 Evolution of Rotation: Pulsars and GRBs 750
28.6.1 Distribution of the Specific Angular Momentum 750
28.6.2 The Rotation of Pulsars 752
28.6.3 GRBs: A Challenging Problem 752
28.6.4 Models for the GRB Progenitors 753

29 Evolution of $Z = 0$ and Very Low Z Stars 755
29.1 Basic Properties and Evolution of $Z = 0$ Stars 755
29.1.1 Differences in the Physics 755
29.1.2 The HR and $\log T_c$ vs. $\log \varrho_c$ Diagrams 756
29.1.3 Low-Mass Stars ($M < 3\,M_\odot$) 757
29.1.4 Intermediate Mass Stars ($3\,M_\odot < M < 10\,M_\odot$) 758
29.1.5 High-Mass Stars ($M > 10\,M_\odot$) 760
29.1.6 Other Properties: Mass Limits and CO Cores 760
29.2 Rotation Effects at $Z = 0$ 761
29.2.1 HR Diagram and Lifetimes 761
29.2.2 Evolution of the Rotation, Final Masses 762
29.3 Rotation Effects in Very Low Z Models 764
29.3.1 Rotational Mass Loss in the First Generations 765
29.3.2 Enrichments by the Winds of the First Generations 766

A Physical and Astronomical Constants 771
A.1 Physical Constants 771
A.2 Some Astronomical Constants 772
A.3 Initial Solar Abundances 772

B Complements on Mechanics and Electromagnetism 773
B.1 Equations of Motion and Continuity 773
B.1.1 Equations of Continuity and of Motion 773
B.1.2 Remarks on Derivatives 774
B.1.3 Vectorial Operators in Spherical Coordinates 774
B.1.4 Viscous Terms 775
B.1.5 Navier–Stokes Equation 776
B.1.6 Equation of Motion with Rotation 777
B.1.7 Geostrophic Motions, Taylor–Proudman Theorem 778
B.2 Maxwell Equations 779
B.3 Statistical Mechanics: Pressure and Energy Density 780
B.3.1 Non-relativistic Particles 781
B.3.2 Relativistic Particles 781
B.4 Expressions of Viscosity, Conductivity and Diffusion 782
B.4.1 Viscosity from Turbulence, Radiation and Plasma 782
B.4.2 Conductivity 784
B.4.3 Diffusion Coefficient 785
B.5 Dimensionless Numbers 785
B.5.1 Reynolds Number 785
B.5.2 Prandtl Number 786

B.5.3 Peclet and Nusselt Numbers 786
B.5.4 The Rossby Number 787
B.6 More on the Physics of Rotation 787
B.6.1 The Angular Velocity in Spherical Functions 787
B.6.2 Rotational Splitting for Non-uniform Rotation 790

C Complements on Radiative Transfer and Thermodynamics 795
C.1 Radiation: Definitions 795
C.1.1 The Quasi-Isotropic Case 798
C.2 Expression of the Heat Changes $dq = dq(P, \varrho)$ 798
C.3 Adiabatic Acoustic Waves 799
C.4 The Entropy of Radiation and Perfect Gas 800
C.4.1 Entropy of Radiation 800
C.4.2 Entropy of a Mixture of Perfect Gas and Radiation 801
C.4.3 Degenerate Gases and Minimum Entropy 802
C.4.4 The Entropy of Mixing 803
C.5 Recalls on Fundamental Statistics 804
C.6 Thermodynamic Equilibrium 805
C.6.1 Reactions with Changes of State 805
C.6.2 Maxwell–Boltzmann Distribution 806

References 807

Index 823